# 9 CHAPTER

# Filters

## 9.1. INTRODUCTION

A filter is a network which passes a particular band of frequencies and attenuates all other frequencies. It is considered as a frequency selective network, because as we know that in resonant circuits of AC circuits, select relatively narrow bands of frequencies and reject other frequencies.

Filter networks are mostly used in communication system for separating the various voice channels in carrier frequency telephone circuits. Filters are also used in instrumentation, telemetering equipments etc, where it is necessary to transmit or attenuate a limited range of frequencies.

Mostly the filters are two types as passive and active filters. A **passive filter** is consisting passive elements e.g. resistor, capacitor, inductor. The cost of passive filter is high because inductors used are heavy and bulky. No gain is possible in a passive filter. The **active filter** is consisting active elements like transistors, op–amps in addition to resistor, capacitor. Voltage, current and power gain can be achieved in active filters. The frabrication of inductors with high quality in integrated circuit (I.C.) technology is not possible, so the resistance capacitance circuit with active device replaces the conventional L–C filter and provides a sharp cut–off in the attenuation band.

In this chapter, we will study various types of filters and its analysis.

## 9.2 PARAMETERS OF FILTER

The various parameters that characterize a typical filter are as follows:

**(*a*) Characteristic Impedance $Z_0$ or $Z_c$.**

The characteristic impedance of filter should be such that the filter may fit into the given line or between the two types of the equipment.

**(*b*) Pass Band**

It is the range of frequencies for which ideal filters have to pass all frequencies without reduction in magnitude is called pass band.

**(*c*) Stop Band**

It is the range of frequencies for which ideal filters have to stop all frequencies. Stop Band is the band where all the frequencies are attenuated through the filters.

**(*d*) Cut–off Frequency $f_c$**

The frequency which separates the pass-band and the stop–band is called the cut–off frequency of the filter.

**(*e*) Units of Attenuation**

The attenuation of a wave filter can be expressed in Nepers, decibels ($dB$) or Bels.

Let $V_i$, $I_i$ and $P_i$ be the input voltage, input current and input power respectively of a filter. Similarly $V_0$, $I_0$ and $P_0$ be the output voltage, output current and output power respectively.

The ratio $\frac{P_o}{P_i}$ represents the power gain and ratio $\frac{V_o}{V_i}$ represents the voltage gain. In the case of $P_i > P_o$ so that $\frac{P_i}{P_o}$ is given by the attenuation.

The various units of attenuation are defined as:

(*i*) A **decibel** is defined as the ten times the common logarithm of the ratio of the input power to the output power.

$$\text{Attenuation in } dB = 10 \log_{10} \frac{P_i}{P_o}$$

The decibel can also be expressed in terms of input current (or voltage) and the output current (or voltage) as

$$\text{Attenuation in } dB = 20 \log_{10} \frac{I_i}{I_o} = 20 \log_{10} \frac{V_i}{V_o}$$

(*ii*) A **Neper** is defined as the natural logarithm of the ratio of input and output quantities.

$$\text{Attenuation in Nepers} = \frac{1}{2} \log_e \frac{P_i}{P_o} = \log_e \frac{I_i}{I_o} = \log_e \frac{V_i}{V_o}$$

(*iii*) A **Bel** is defined as the common logarithm of the ratio of input and output quantities.

$$\text{Attenuation is Bels} = \log_{10} \frac{P_i}{P_o} = 2 \log_{10} \frac{I_i}{I_o} = 2 \log_{10} \frac{V_i}{V_o}.$$

We can obtain a relation between these units as

Attenuation in $dB$ = 8.686 × Attenuation in Nepers

= 10 × Attenuation in Bels.

## 9.3 CLASSIFICATION OF FILTERS

Based on frequency characteristics, the filters are classified as follows:

**(*a*) Low pass filters**

Those filters which are pass the frequency upto the cut–off frequency $f_c$ and the frequency above it are rejected. Fig. 9.1 shows the attenuation characteristic of an ideal low pass filter. Therefore, the frequency range of the pass band or transmission band of low pass filter is $o$ to $f_c$ and the range of frequency of the stop band or attenuation band or rejected frequency is above the $f_c$.

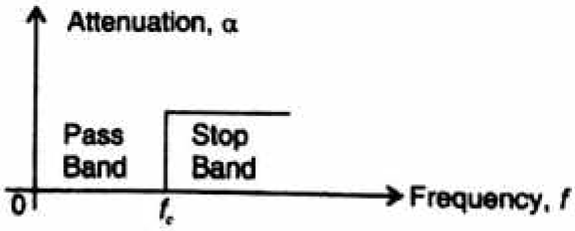



**Fig. 9.1.** Characteristics of an ideal low pass filter.

**(*b*) High Pass Filters**

Those filters which are pass the frequency above the cut-off frequency $f_c$ and the frequency below it are rejected. Fig. 9.2. shows the attenuation characteristic of an ideal high pass filter. Therefore, the frequency range of the pass band or transmission band of high pass filter is above the $f_c$ and the range of frequency of the stop band or attenuation band or rejected frequency is $o$ to $f_c$.

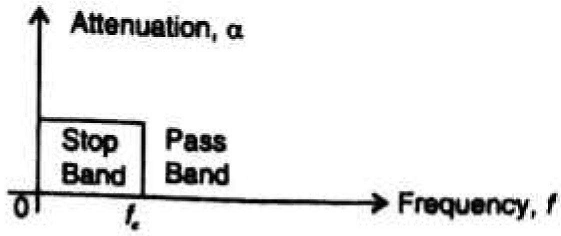



**Fig. 9.2.** Characteristics of an ideal high pass filters

**(*c*) Band Pass Filters**

Those filters which are pass the frequencies between two designed cut–off frequencies and reject all other frequencies. Fig 9.3 shows the attenuation characteristic of an ideal band pass filters. A band pass filter has two cut off frequencies *i.e.* $f_{c1}$ is called as the lower cut off frequency while $f_{c2}$ is called the upper cut-off frequency. The range of frequency for pass band is $f_{c2} - f_{c1}$.

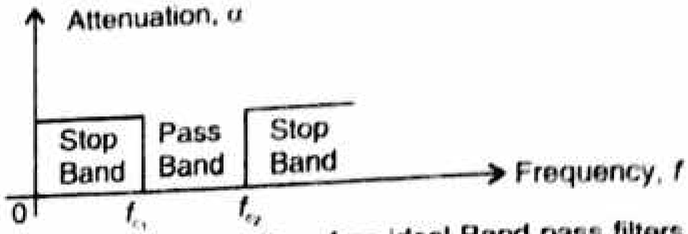



**Fig. 9.3.** Characteristics of an ideal Band pass filters.

**(*d*) Band stop or Band Elimination or Band Reject Filters**

Those filters which are reject the frequencies between two designed cut-off frequencies and pass all other frequencies. Fig. 9.4 shows the attenuation characteristic of an ideal band stop filter. The range of frequency for stop band is $f_{c2} - f_{c1}$.

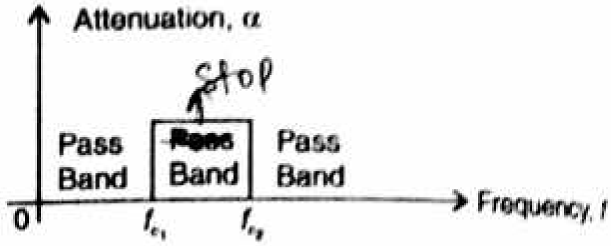



**Fig. 9.4.** Characteristics of an ideal band pass filters.

## 9.4 BLOCK DIAGRAM REPRESENTATION OF THE FILTERS

Fig. 9.5 (*a*) and (*b*) represents the block diagram of low pass and high pass filters respectively.

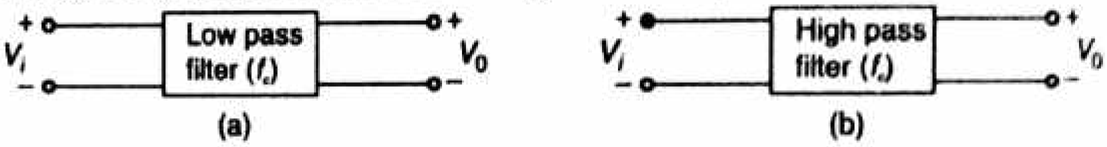



**Fig. 9.5.** Block diagram representation of the filters.

A band pass filter can be obtained by the series or cascade connection of low pass and high pass filter as shown in Fig. 9.6. The cut-off frequency of low pass filter is $f_{c_2}$ and that of high pass filter is $f_{c_1}$ where $f_{c_2} > f_{c_1}$.

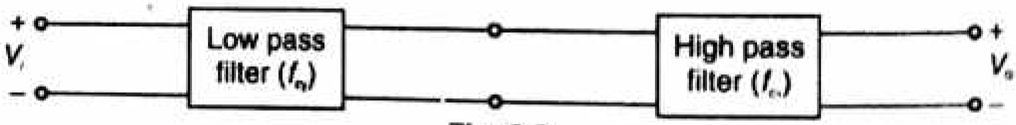



**Fig. 9.6**

A band-stop filter is obtained by parallel connection of a low-pass filter and high pass filter as shown in Fig. 9.7. The cut-off frequency of low pass filter is $f_{c_1}$ and that of high pass filter is $f_{c_2}$ where $f_{c_2} > f_{c_1}$.

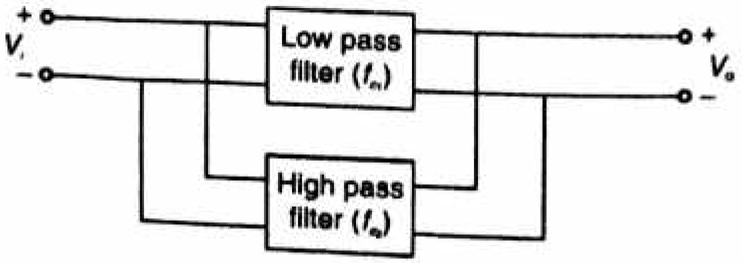



**Fig. 9.7**

Mathematically, we can define the band pass filter is intersection of low-pass and high pass filters. *i.e.*

$$B.P = (L.P.) \cap (H.P.)$$

And the band – Stop filter is union of low pass and high pass filters *i.e.*

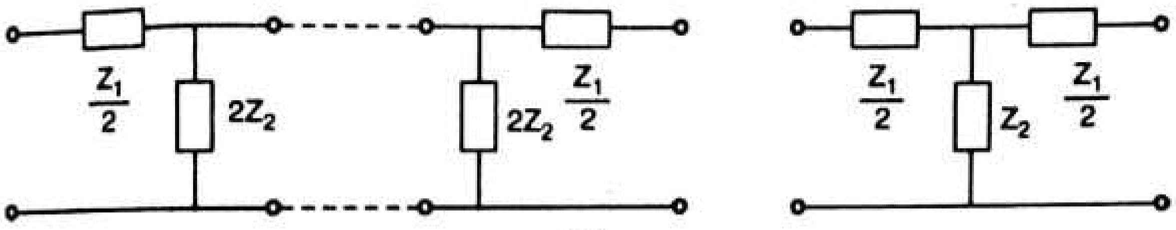
$$B.S = (L.P.) \cup (H.P.)$$

**Table 9.1** Shows the summarizing of filter characteristics

| Type of Filter | Pass Band | Attenuation or stop band |
|---|---|---|
| Low – pass | 0 to $f_c$ | $f_c$ to $\infty$ |
| High – pass | $f_c$ to $\infty$ | 0 to $f_c$ |
| Band – pass | $f_{c_1}$ to $f_{c_2}$ | 0 to $f_{c_1}$ and $f_{c_2}$ to $\infty$ |
| Band – Stop | 0 to $f_{c_1}$ and $f_{c_2}$ to $\infty$ | $f_{c_1}$ to $f_{c_2}$ |

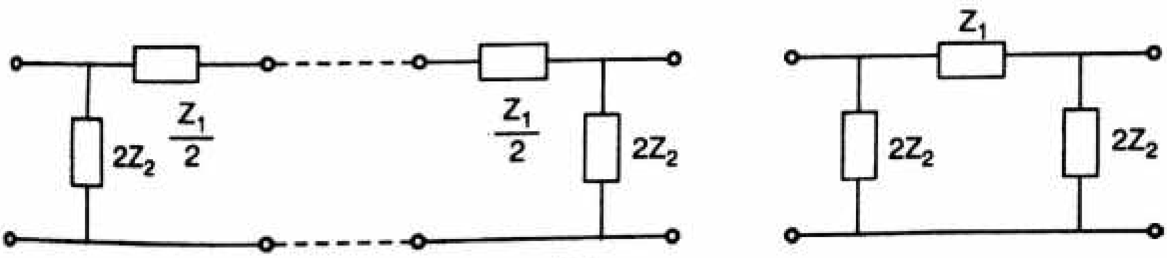
## 9.5 FILTER NETWORKS

There should be no attenuation in pass band for an ideal filter. This can only happen when filter is pure reactive or dissipationless. The filter networks are made up of symmetrical $T$ and $\pi$ networks. Both the $T$ and $\pi$ networks can be considered as combinations of asymmetrical L networks as shown in Fig. 9.8($a$) and ($b$) respectively.




(a)

**(*a*)** Symmetrical *T*–network

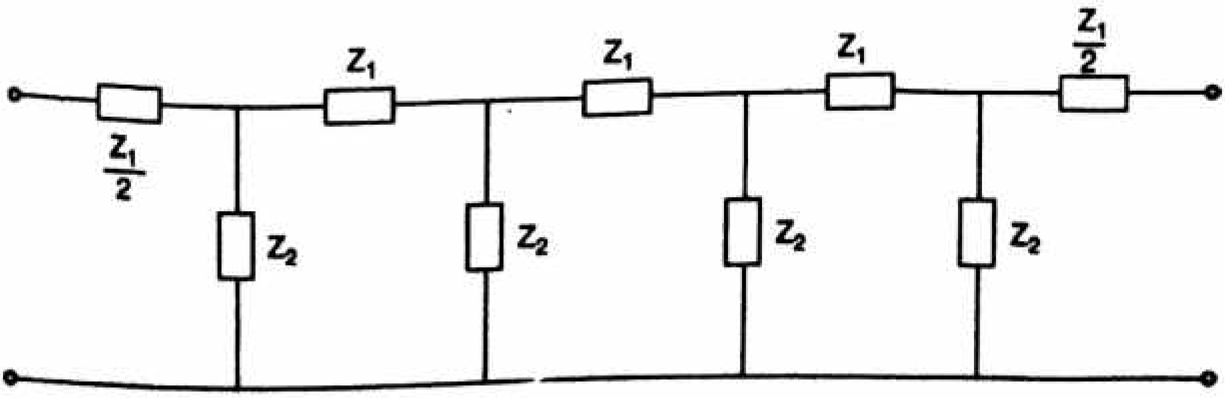



(b)

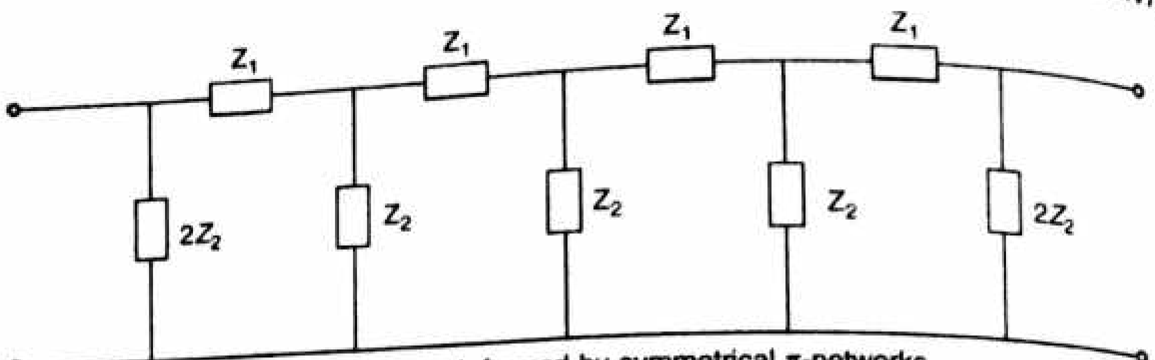
**(*b*)** Symmetrical $\pi$ – network

**Fig. 9.8.** Filter networks

Filter networks are consisting of ladder network. A combination of several $T$ and $\pi$ networks constitutes a ladder networks. The common forms of the ladder networks formed by symmetrical $T$ and symmetrical $\pi$–networks are shown in Fig. 9.9 ($a$) and ($b$) respectively. It can be seen that both networks are identical except at the ends.




**(*a*)** Ladder network formed by symmetrical *T*–networks.




(*b*) Ladder network formed by symmetrical $\pi$-networks

**Fig. 9.9**

## 9.6 CHARACTERISTICS OF FILTER NETWORKS

The study of the characteristics of any filter requires the following:

(*a*) Characteristic impedance ($Z_0$)

(*b*) Propagation constant ($\gamma$)

(*c*) Attenuation ($\alpha$) and

(*d*) Phase shift ($\beta$)

### 9.6.1 Characteristic Impedance ($Z_0$)

As we know that for a two–part network, if the image impedances at input port and output port are equal to each other, then the image impedance is called the characteristic or the iterative impedance $Z_0$.

**(*a*) For T–Network**

Consider a symmetrical *T*– network with input impedance $Z_0$, is terminated in same impedance $Z_0$ as shown in Fig. 9.10.

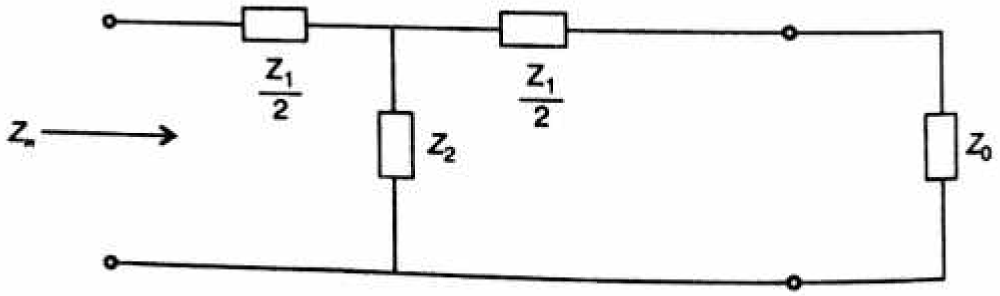



**Fig. 9.5** Symmetrical T–network terminated in $Z_0$

The input impedance is given as

$$Z_{in} = Z_0 = \frac{Z_1}{2} + \left[Z_2 \parallel \left(\frac{Z_1}{2} + Z_0\right)\right]$$

or

$$Z_0 = \frac{Z_1}{2} + \frac{Z_2 \cdot \left(\frac{Z_1}{2} + Z_0\right)}{Z_2 + \frac{Z_1}{2} + Z_0}$$

$$= \frac{Z_1^2 + 2Z_1 Z_2 + 2Z_1 Z_0 + 2Z_1 Z_2 + 4Z_2 Z_0}{2(Z_1 + 2Z_2 + 2Z_0)}$$

or

$$2 Z_0 (Z_1 + 2Z_2 + 2Z_0) = Z_1^2 + 4Z_1Z_2 + 2Z_1Z_0 + 4Z_2Z_0$$

$$4Z_0^2 = Z_1 + 4Z_1Z_2$$

or $$Z_0 = \frac{Z_1^2}{4} + Z_1 Z_2$$

Therefore, the characteristic impedance of a symmetrical T-network is

$$\boxed{Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}}$$

**$Z_{0T}$ in term of open ($Z_{0c}$) and short circuit ($Z_{sc}$) impedance:**

For open circuit impedance, the network becomes as shown in Fig. 9.11.

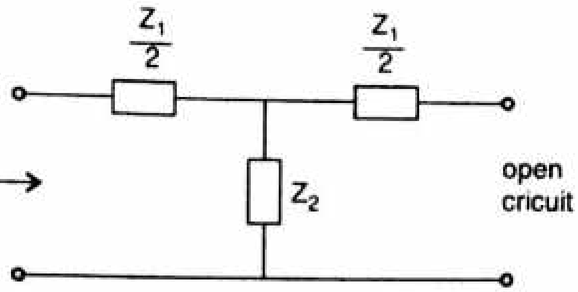



Fig. 9.11

$$\therefore \quad Z_{0c} = \frac{Z_1}{2} + Z_2$$

or $$Z_{0c} = \frac{1}{2}(Z_1 + 2Z_2)$$

For short circuit impedance, the network becomes as shown in Fig. 9.12.

$$\therefore \quad Z_{sc} = \frac{Z_1}{2} + \left[\frac{Z_1}{2} \middle\| Z_2\right]$$

$$= \frac{Z_1}{2} + \frac{\frac{Z_1}{2}.Z_2}{\frac{Z_1}{2} + Z_2}$$

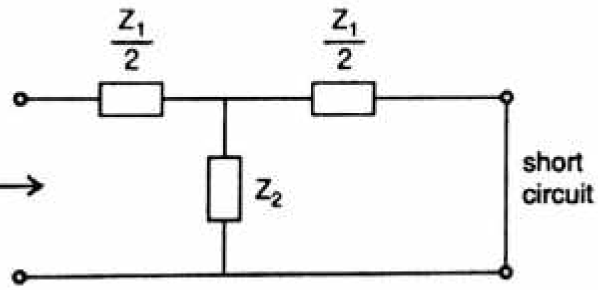



Fig. 9.12

or $$Z_{sc} = \frac{Z_1^2 + 4Z_1Z_2}{2(Z_1 + 2Z_2)}$$

Therefore, $$Z_{0c} \times Z_{sc} = \frac{Z_1}{4} + Z_1 Z_2 = Z_{0T}^2$$

or $$Z_{0T} = \sqrt{\boldsymbol{Z_{0c}.Z_{sc}}}$$

**(b) For $\pi$-network**

Consiser a symmetrical $\pi$-network with input impedance $Z_0$, is terminated in same impedance $Z_0$ as shown in Fig. 9.13.

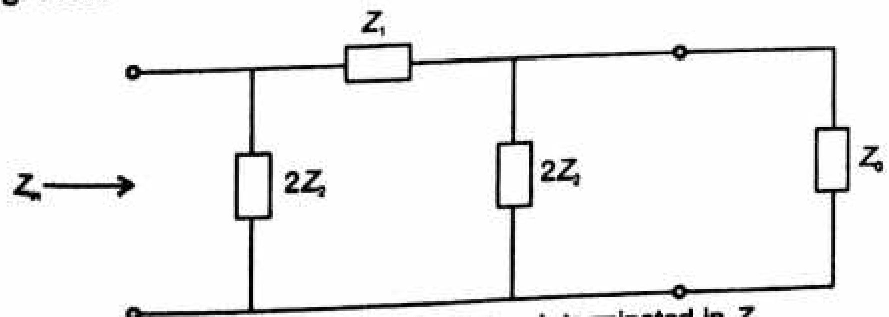



Fig. 9.13. Symmetrical $\pi$-network terminated in $Z_0$.

The input impedance is given as

$$Z_{in} = (2Z_2) \| [(2Z_2 \| Z_0) + Z_1]$$

$$= (2Z_2) \left\| \left( \frac{2Z_2.Z_0}{2Z_2 + Z_0} + Z_1 \right) \right.$$

$$= \frac{2Z_2 \cdot \left( \dfrac{2Z_2 Z_0}{2Z_2 + Z_0} + Z_1 \right)}{2Z_2 + \dfrac{2Z_2 Z_0}{2Z_2 + Z_0} + Z_1}$$

or $$Z_0 = \frac{2Z_2(2Z_2 Z_0 + Z_1 Z_0 + 2Z_1 Z_2)}{2Z_2 Z_0 + Z_1 Z_0 + 2Z_1 Z_2 + 2Z_2 Z_0 + 4Z_2^2}$$

or $$2Z_2 Z_0^2 + Z_1 Z_0^2 + 2 Z_1 Z_2 Z_0 + 2 Z_2 Z_0^2 + 4 Z^2{}_2 Z_0 = 4 Z^2{}_2 Z_0 + 2 Z_1 Z_2 Z_0 + 4 Z_1 Z^2{}_2$$

or $$4Z_2 Z_0^2 + Z_1 Z_0^2 = 4 Z_1 Z^2{}_2$$

or $$Z_0^2 = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2}$$

Therefore, the characteristic impedance of symmetrical $\pi$-network is

$$Z_{0\pi} = \frac{2Z_1 Z_2}{\sqrt{Z_1^2 + 4Z_1 Z_2}} = \frac{Z_1 Z_2}{\sqrt{\dfrac{Z_1^2}{4} + Z_1 Z_2}}$$

or $$\boxed{\mathbf{Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{\dfrac{Z_1^2}{4} + Z_1 Z_2}}}}$$

**$Z_{0\pi}$ in terms of open circuit ($Z_{0c}$) and short circuit ($Z_{sc}$) impedance:**

For open circuit impedance, the network becomes as shown in Fig. 9.14.

$\therefore$ $$Z_{0c} = (2Z_2) \| (Z_1 + 2Z_2)$$

or $$Z_{0c} = \frac{2Z_2.(Z_1 + 2Z_2)}{Z_1 + 4Z_2}$$

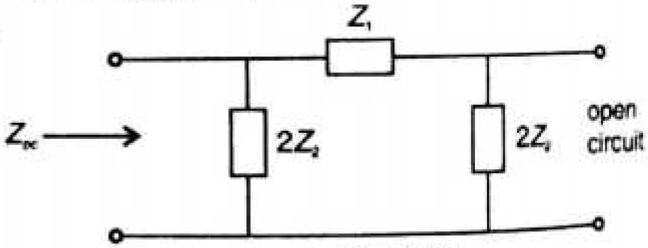



Fig. 9.14

For short circuit impedance, the network becomes as shown in Fig. 9.15, since $2Z_2$ is also short circuited.

$\therefore$ $$Z_{sc} = 2Z_2 \| Z_1$$

or $$Z_{sc} = \frac{2Z_1 Z_2}{Z_1 + 2Z_2}$$

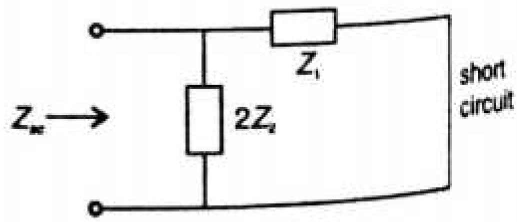



Fig. 9.15

Therefore, $$Z_{0c} \times Z_{sc} = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2} = Z_{0\pi}^2$$

or $$\boxed{\mathbf{Z_{0\pi} = \sqrt{Z_{oc}.Z_{sc}}}}$$

Thus, we can also get

$$\boxed{\mathbf{Z_{0T}.Z_{0\pi} = Z_1 Z_2}}$$

### 9.6.2 Propagation constant ($\gamma$)

The propagation constant $\gamma$ of the network is given by

$$\gamma = \log_e \left( \frac{I_1}{I_2} \right)$$

**For *T*-network**

Consider a *T*-network is terminated with characteristic impedance $Z_0$ as shown in Fig. 9.16.

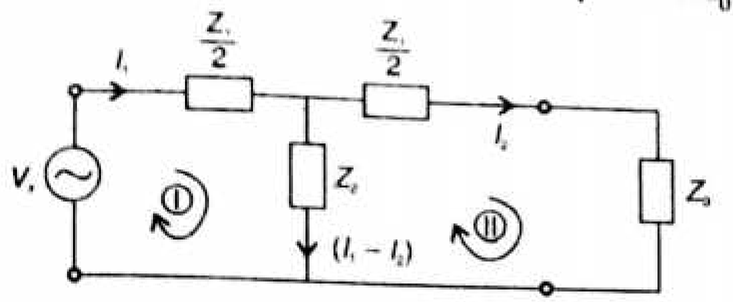



**Fig. 9.16**

By KVL, we can write

**In loop-I:** $\frac{Z_1}{2} I_1 + Z_2 (I_1 - I_2) - V_s = 0$ ... (1)

**In loop-II:** $\left(Z_0 + \frac{Z_1}{2}\right) I_2 - Z_2 (I_1 - I_2) = 0$ ... (2)

or
$$\frac{I_1}{I_2} = \frac{\frac{Z_1}{2} + Z_2 + Z_0}{Z_2} = e^{\gamma} \text{ (From definition)}$$

or
$$\frac{Z_1}{2} + Z_2 + Z_0 = Z_2\, e^{\gamma}$$

or
$$Z_0 = (e^{\gamma} - 1)\, Z_2 - \frac{Z_1}{2} \qquad \text{... (3)}$$

We know that the characteristic impedance of a symmetrical *T*-network is given by

$$Z_{0T} = \sqrt{\frac{Z_1}{4} + Z_1 Z_2} \qquad \text{... (4)}$$

Squaring equations (3) and (4), and subtracting equation (4) from equation (3), we get

$$Z_2^2 (e^{\gamma} - 1)^2 + \frac{Z_1^2}{4} - (e^{\gamma} - 1)\, Z_1 Z_2 - \frac{Z_1^2}{4} - Z_1 Z_2 = 0$$

$$Z_2^2 (e^{\gamma} - 1)^2 - Z_1 Z_2 (e^{\gamma} - 1 + 1) = 0$$

$$Z_2 (e^{\gamma} - 1)^2 - Z_1\, e^{\gamma} = 0$$

$$(e^{\gamma} - 1)^2 = \frac{Z_1 e^{\gamma}}{Z_2}$$

$$e^{2\gamma} + 1 - 2e^{\gamma} = \frac{Z_1}{Z_2} e^{\gamma}$$

or
$$e^{\gamma} + e^{-\gamma} - 2 = \frac{Z_1}{Z_2}$$

Dividing both the sides by 2, we get

$$\frac{e^{\gamma} + e^{-\gamma}}{2} = 1 + \frac{Z_1}{2Z_2}$$

$$\boxed{\cos h\gamma = 1 + \frac{Z_1}{2Z_2}}$$

In other forms we can find as follows:

$$\sin h\gamma = \sqrt{\cos h^2\gamma - 1} = \sqrt{\left(1+\frac{Z_1}{2Z_2}\right)^2 - 1}$$

$$= \sqrt{\left(\frac{Z_1}{2Z_2}\right)^2 + \frac{Z_1}{Z_2}}$$

$$= \frac{1}{Z_2}\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

or $$\boxed{\sin h\,\gamma = \frac{Z_{0T}}{Z_2}}$$ [From equations (4)]

Therefore, $$\boxed{\tan h\,\gamma = \frac{Z_{0T}}{Z_2\left(1+\frac{Z_1}{2Z_2}\right)} = \frac{Z_{0T}}{\left(Z_2+\frac{Z_1}{2}\right)}}$$

We know that, $$Z_{0c} = Z_2 + \frac{Z_1}{2}$$

and $$Z_{0T} = \sqrt{Z_{oc} Z_{sc}}$$

Therefore, $$\boxed{\tan h\gamma = \frac{\sqrt{Z_{oc}.Z_{sc}}}{Z_{oc}} = \sqrt{\frac{Z_{sc}}{Z_{oc}}}}$$

Also $$\sin h\,\frac{\gamma}{2} = \sqrt{\frac{1}{2}(\cos h\gamma - 1)} = \sqrt{\frac{1}{2}\left(1+\frac{Z_1}{2Z_2}-1\right)} = \sqrt{\frac{Z_1}{4Z_2}}$$

**(b) For π-network:**

The propagation constant of a symmetrical π-network is the same as that for a symmetrical *T*-network *i.e.*

$$\boxed{\cos h\,\gamma = 1 + \frac{Z_1}{2Z_2}}$$

### 9.6.3 Classification of Pass-band and stop-band

For designing any filter the following requirements should be fulfilled.

*(i)* A filter should pass (or transmit) desired frequencies without any attenuation. It means attenuation (α) should be zero in the pass band.

*(ii)* It should attenuate or completely stop all undesired frequencies. It means in the stop band, 'α' should be high.

The propagation constant is a function of frequency is given by

$$\gamma = \alpha + j\beta$$

Where, α = attenuation and β = phase shift

The propagation constant should provide the information of ability of the filter to fulfill basic requirements.

It α = o, then output current ($I_2$) is equal to the input current ($I_1$). This is because no attenuations is provided by the filter network. So there exists only phase shift β.

If $\alpha$ is positive ($\alpha > 0$) then the filter network provides attenuation. In this case $I_1 > I_2$ and it indicates the operation of band.

For symmetrical $T$ or $\pi$-network, the propagation constant is given by

$$\sin h\,\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

or
$$\sin h\left(\frac{\alpha + j\beta}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}}$$

or
$$\sin h\frac{\alpha}{2}.\cos\frac{\beta}{2} + j\cos h\frac{\alpha}{2}.\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad \text{... (1)}$$

($\because$ $\sin(A + B) = \sin A\cos B + \cos A\sin B$)

**Consider following cases:**

**Case I: When $Z_1$ and $Z_2$ are of same type of reactances.**

In this case $\frac{Z_1}{4Z_2}$ is real and positive.

Therefore, imaginary part of equation (1) must be zero.

$$\therefore \quad \cos h\,\frac{\alpha}{2}.\sin\frac{\beta}{2} = 0 \quad \text{... (2)}$$

and
$$\sin h\,\frac{\alpha}{2}.\cos\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad \text{... (3)}$$

Equation (2) can be satisfied if

$$\frac{\beta}{2} = n\pi, \text{ where } n = 0, 1, 2, 3, \text{.......}$$

then $\cos\frac{\beta}{2} = 1$ and $\sin h\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$ (From equation (3))

Therefore,
$$\boxed{\alpha = 2\sin h^{-1}\sqrt{\frac{Z_1}{4Z_2}}}$$

This gives high value of attenuation. Thus the condition $\left|\frac{Z_1}{4Z_2}\right| > 0$, gives attenuation band.

**Case II: When $Z_1$ and $Z_2$ are of opposite type of reactances.**

In this case $\frac{Z_1}{4Z_2}$ is real and negative.

Therefore, $\sqrt{\frac{Z_1}{4Z_2}}$ will be imaginary and real part of equation (1) must be zero.

$$\therefore \quad \sin h\,\frac{\alpha}{2}\cos\frac{\beta}{2} = 0 \quad \text{... (4)}$$

and
$$\cos h\,\frac{\alpha}{2}\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad \text{... (5)}$$

Equations (4) and (5) must be satisfied simultaneously by $\alpha$ and $\beta$. Equation (4) may be satisfied when $\alpha = 0$, or when $\beta = \pi$.

**Condition : (i) – When $\alpha = 0$.**

Then from equation (5), we get

$$\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

But $\frac{Z_1}{4Z_2}$ must be negative.

Therefore, $\quad -1 \le \frac{Z_1}{4Z_2} \le 0$

and $$\boxed{\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}} \qquad ...(6)$$

In this case, since $\alpha = 0$, it gives pass band. The phase shift introduced in the pass band is given by equation (6).

**Condition : (ii) when $\beta = \pi$,**

Therefore, $\cos\frac{\beta}{2} = 0$, and $\sin\frac{\beta}{2} = \pm 1$

Then from equation (5), we get

$$\cos h\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

or $$\boxed{\alpha = 2\cos h^{-1}\sqrt{\frac{Z_1}{4Z_2}}} \qquad ...(7)$$

Since $\alpha \neq 0$, it gives attenuation (stop) band.

**Cut-off Frequency**

The frequency at which the network changes from a pass-band to stop-band and vice-versa is called the cut-off frequency.

The cut-off frequencies can be obtained by using,

$$-1 < \frac{Z_1}{4Z_2} < 0$$

$\therefore$ $$\frac{Z_1}{4Z_2} = 0 \qquad \text{or} \qquad Z_1 = 0$$

and $$\frac{Z_1}{4Z_2} = -1 \qquad \text{or} \qquad Z_1 + 4Z_2 = 0$$

Where $Z_1$ and $Z_2$ are opposite type of reactances.

## 9.7 SUMMARY OF PARAMETERS FOR FILTER NETWORKS

The summary of parameters for T and $\pi$-Filter networks are summarize as follows:

1. Characteristic impedance ($Z_0$):

For a T-network: $$Z_{0T} = \sqrt{Z_1Z_2\left(1+\frac{Z_1}{4Z_2}\right)}$$

For a π-network: $$Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}}$$

$$Z_{0T} \cdot Z_{0\pi} = Z_1 Z_2$$

2. **Propagation Constant (γ):**

The propagation constant is same for T and π-network and is given by

$$\cosh \gamma = 1 + \frac{Z_1}{2Z_2}$$

3. **Attenuation Constant (α):**

$$\alpha = 0; \quad \text{in pass-band}$$

$$\alpha = 2 \cosh^{-1} \sqrt{\left(\frac{Z_1}{4Z_2}\right)}; \text{ in stop-band.}$$

4. **Phase Constant (β):**

$$\beta = 2 \sin^{-1} \sqrt{\left|\frac{Z_1}{4Z_2}\right|}; \text{ in pass-band.}$$

$$\beta = n\pi,\ n = 0, 1, 2, \ldots; \text{ in stop-band.}$$

5. **Cut-off Frequency:**

$$Z_1 = 0 \text{ and } Z_1 + 4 Z_2 = 0$$

## 9.8 TYPE OF FILTERS

Depending on the relationship between $Z_1$ and $Z_2$, filters are classified as:

(a) **Constant -*K* or Prototype Filters:**

In this type of filters, $Z_1$ and $Z_2$ related as

$$Z_1 Z_2 = K^2 = R_0^2$$

Where $K$ is a real constant, independent of frequency, *i.e.*, a resistance. $K$ is also known as design impedance $R_0$ of the filter.

The constant –$K$ type filter is also known as the prototype because other more complex networks can be derived from it.

(b) ***m*-derived Filters:**

In this type of filters, $Z_1$ and $Z_2$ related as

$$Z_1 Z_2 \neq K^2$$

### 9.8.1 Constant –*K* Low Pass Filters

*T* and π-network for constant -*K* low pass filter shown in Fig. 9.17 (*a*) and (*b*) respectively.

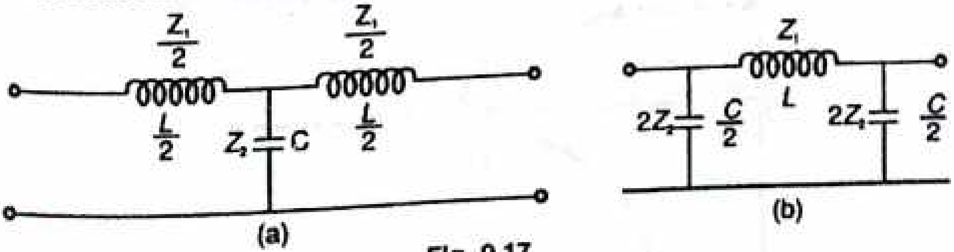



**Fig. 9.17**

For both *T* and π-networks, we have

$$Z_1 Z_2 = K^2 = R_0^2$$

For both $T$ and $\pi$-network filters:

If series impedance $Z_1 = jwL$ and shunt impedance $Z_2 = \dfrac{1}{j\omega C}$, then

$$Z_1 Z_2 = \frac{L}{C} = K^2 = R^2_0 \qquad \text{(which is independent of frequency)}$$

Therefore, design impedance is given by

$$\boxed{R_0 = \sqrt{\frac{L}{C}}}$$

(*i*) **Cut-off frequency:**

The cut-off frequency is given by

$$Z_1 = 0$$

or $$j\omega_c L = 0$$

or $$\omega_c = 0 \qquad \text{or} \qquad \boxed{f_c = 0}$$

and $$Z_1 + 4 Z_2 = 0$$

or $$j\omega_c L + 4.\frac{1}{j\omega_c C} = 0$$

or $$\omega_c = \frac{2}{\sqrt{LC}} \qquad (\because j^2 = -1)$$

or $$\boxed{f_c = \frac{1}{\pi\sqrt{LC}}} \qquad (\because \omega_c = 2\pi f_c)$$

Therefore, the pass-band of constant $-K$ low pass filter extends from 0 to $\dfrac{1}{\pi\sqrt{LC}}$.

The graphical representation of constant $-K$ low pass filter is shown in Fig. 9.18.

Since, $Z_1 = j\omega_c L = j_2\pi f_c L$ and $Z_2 = \dfrac{1}{j\omega_c C} = \dfrac{1}{j_2\pi f_c C}$ therefore reactances $Z_1$ and $Z_2$ vary with frequency as linear and rectangular hyperbola as shown in Fig. 9.18. The cut-off frequency at the intersection of the curve $Z_1$ and $-4Z_2$ is indicated as $f_c$. All the frequencies above $f_c$ lie in a stop or attenuation band. Thus, the network is called a low pass filters.

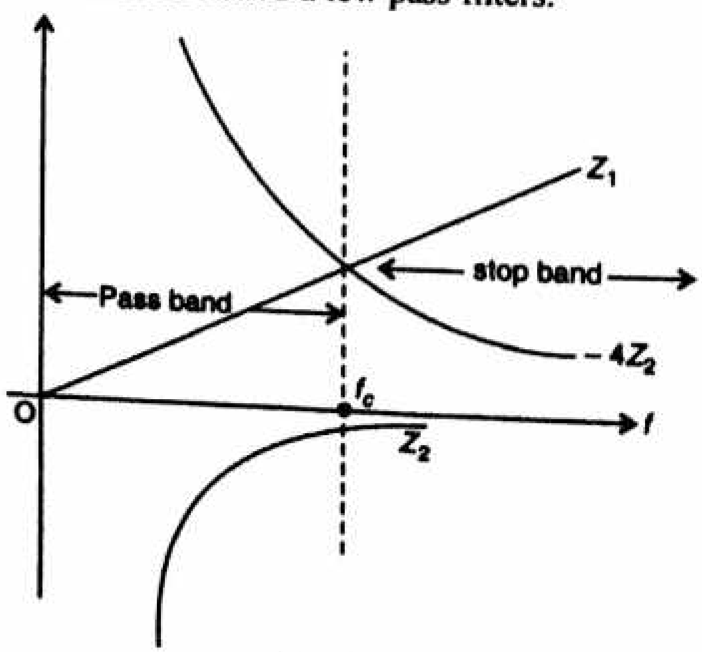



Fig. 9.18.

(ii) **Attenuation constant:**

In pass-band, $\alpha = 0$

and in stop-band, $\alpha = 2\cosh^{-1}\sqrt{\dfrac{Z_1}{4Z_2}}$

$\because$ $Z_1 = j\omega L$ and $Z_2 = \dfrac{1}{j\omega C}$

$\therefore$ $$\alpha = 2\cosh^{-1}\sqrt{\frac{\frac{j\omega L}{4}}{j\omega C}} = 2\cosh^{-1}\sqrt{\frac{\omega^2 LC}{4}} = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$$

(iii) **Phase constant:**

In stop-band, $\beta = \pi$

and in pass-band, $\beta = 2\sin^{-1}\sqrt{\dfrac{Z_1}{4Z_2}} = 2\sin^{-1}\left(\dfrac{f}{f_c}\right)$

The variation of attenuation constant $\alpha$ and phase constant $\beta$ with frequency is shown in Fig. 9.19.

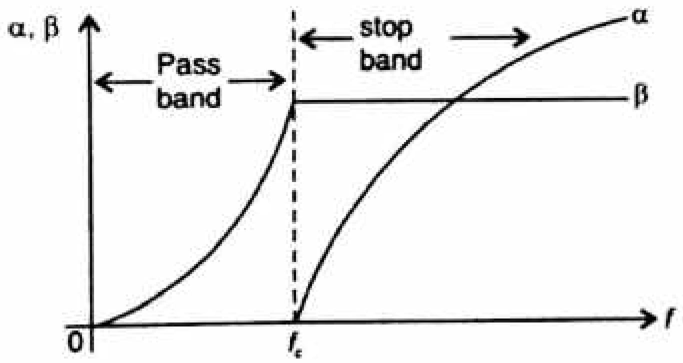



Fig. 9.19. Variation of attenuation constant and phase constant with frequency.

(iv) **Characteristic Impedance:**

The characteristic impedance for $T$-network filter is given by

$$Z_{0T} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}$$

Putting $Z_1 = j\omega L$ and $Z_2 = \dfrac{1}{j\omega C}$, then we get

$$Z_{0T} = \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)}$$

$$= \sqrt{\frac{L}{C}\left[1 - \left(\frac{f}{f_c}\right)^2\right]}$$

or $$Z_{0T} = R_0\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

In the pass-band, $f < f_c$, so that $Z_{0T}$ is real.

In the stop-band, $f > f_c$, so that $Z_{0T}$ is imaginary, and if $f = f_c$, $Z_{0T} = 0$.

The characteristic impedance for $\pi$-network is given by

$$Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

Putting $Z_1 = j\omega L$ and $Z_2 = \frac{1}{j\omega C}$, then we get

$$Z_{0\pi} = \sqrt{\frac{\frac{L}{C}}{1 - \frac{\omega^2 LC}{4}}} = \frac{R_0}{\sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

In the pass-band, $f < f_c$, so that $Z_{0\pi}$ is real.

In the stop-band, $f > f_c$, so that $Z_{0\pi}$ is imaginary, and if $f = f_c$, $Z_{0\pi} = \infty$

The variation of characteristic impedances $Z_{0T}$ and $Z_{0\pi}$ with frequency is shown in Fig. 9.20.

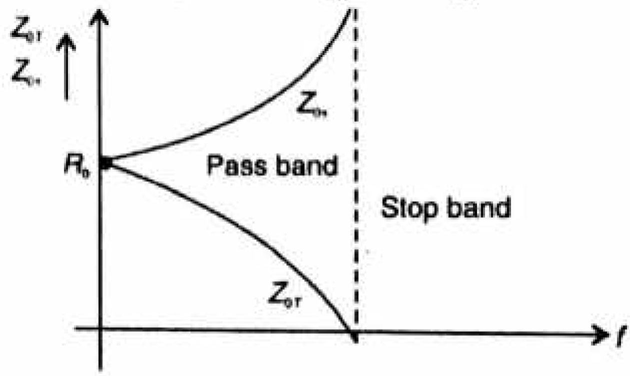



**Fig. 9.20** The variation of $Z_{0T}$ and $Z_{0\pi}$ with f for constant -$K$ type $T$ and $\pi$ networks low pass filters.

**(v) Filter component values.**

We have $Z_1 Z_2 = K^2 = R^2_0$

Putting $Z_1 = j\omega L$ and $Z_2 = \frac{1}{j\omega C}$, then we get

$$R^2_0 = \frac{L}{C} \quad \text{... (1)}$$

and

$$f_c = \frac{1}{\pi\sqrt{LC}} \quad \text{... (2)}$$

From equations (1) and (2), we get

Series inductance; $L = \frac{R_0}{\pi f_c}$ and shunt capacitance, $C = \frac{1}{\pi f_c R_0}$

**Example. 9.1** *Design a prototype low pass filter as T and $\pi$-networks for the characteristic impedance of $R_0 = 500\ \Omega$ and cut-off frequency of 2000 Hz.*

**Solution.**

**Step 1:** To obtain the values of $L$ and $C$.

Given, $f_c = 2000$ Hz, $R_0 = 500\ \Omega$

We know, that, $L = \frac{K}{\pi f_c}$ (where $= K = R_0$)

$$\therefore \quad L = \frac{500}{3.14 \times 2000} = 79.57 \text{ mH}$$

and $$C = \frac{1}{\pi f_c K}$$

$$\therefore \quad C = \frac{1}{3.14 \times 2000 \times 500} = 0.3183 \ \mu\text{F}$$

**Step II: To obatin $T$ and $\pi$-networks for a prototype low pass filter:**

The T-network is shown in Fig. E 9.1 (*a*).

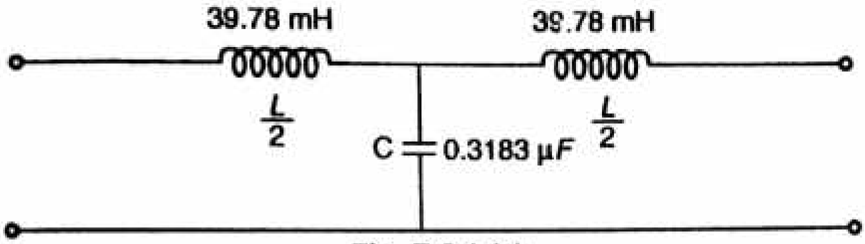



**Fig. E 9.1 (*a*).**

The $\pi$-network is shown in Fig. E 9.2(*b*).

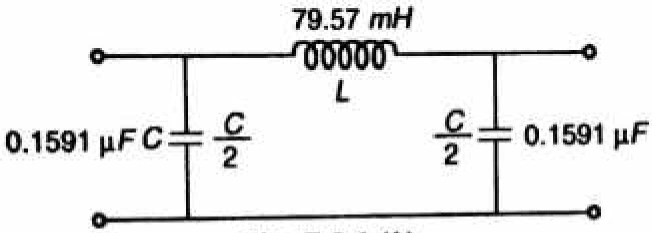



**Fig. E 9.1 (*b*).**

**Example. 9.2.** *Design a low pass filter as $\pi$ and T-networks having a cut-off freqency $f_c$ = 1000 Hz to operate with a terminated load of resistance 200 Ω. Also find the frequency at which this filter offers attenuation of 19.1 dB.*

**Solution.**

**Step I: To design low pass filter as $\pi$ and $T$-network.**

Given, $f_c = 1000$ Hz, $R_0 = 200\ \Omega = K$.

We know that, $$L = \frac{K}{\pi f_c}$$

$$\therefore \quad L = \frac{200}{3.14 \times 1000} = 63.66 \text{ mH}$$

and $$C = \frac{1}{\pi f_c K}$$

$$\therefore \quad C = \frac{1}{3.14 \times 1000 \times 200} = 1.59 \ \mu F$$

The $\pi$ and $T$-networks of low pass filter are shown in Fig. E 9.2(*a*) and (*b*) respectively.

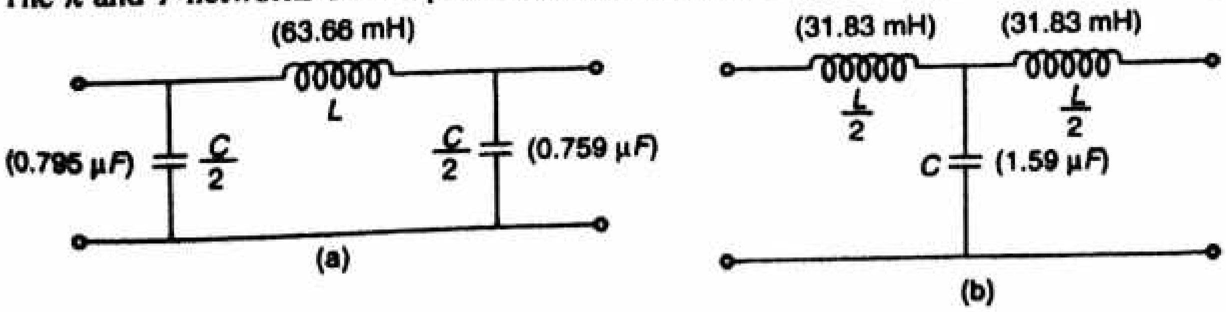



**Fig. E 9.2.**

**Step II: To determine the frequency.**

We know that, attenuation in $dB$ = 8.686 × attenuation in nepers

$$\therefore \quad \text{attenuation in nepers} = \frac{19.1}{8.686} = 2.2 \text{ nepers.}$$

We know that, for Low pass filters: $\alpha = 2 \cos h^{-1}\left(\frac{f}{f_c}\right)$

$$\therefore \quad 2.2 = 2 \cos h^{-1}\left(\frac{f}{1000}\right)$$

or
$$\frac{f}{1000} = \cos h\,(1.1)$$

or
$$f = 1.67 \text{ KHz.}$$

**Example. 9.3** *Determine the nominal characteristic impedance or load resistance and the cut-off frequency for the low pass filter as shown in Fig. E 9.3.*

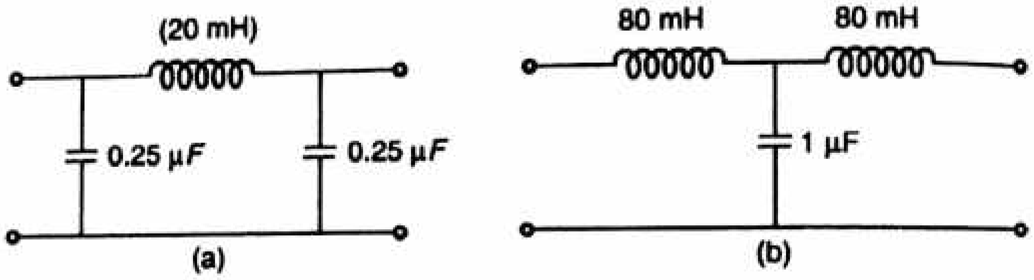



**Fig. E 9.3.**

**Solution.**

**Step I: To obtain the load resistance and the cut-off frequency for the π-network.**

Given;
$$\frac{C}{2} = 0.25\ \mu F \quad \text{or } C = 0.5\ \mu F$$
$$L = 20 \text{ mH}$$

We know that, load resistance or nominal characteristic impedance is given by $K = \sqrt{\frac{L}{C}}$

$$\therefore \quad K = \sqrt{\frac{20\times10^{-3}}{0.5\times10^{-6}}} = \mathbf{200\ \Omega}$$

$$\because \quad \text{Cut-off frequency, } f_c = \frac{K}{\pi L} \left(\text{or } f_c = \frac{1}{\pi KC}\right)$$

$$\therefore \quad f_c = \frac{200}{3.14\times20\times10^{-3}} = \mathbf{3183\ Hz.}$$

**Step II: To obtain the load resistance and the cut-off frequency for the *T*-network.**

Given,
$$C = 1\ \mu F, \quad \frac{L}{2} = 80 \text{ mH or } L = 160 \text{ mH.}$$

$$\therefore \quad K = \sqrt{\frac{L}{C}} = \sqrt{\frac{160\times10^{-3}}{1\times10^{-6}}} = \mathbf{400\ \Omega}$$

and
$$f_c = \frac{K}{\pi L} = \frac{400}{3.14\times160\times10^{-3}} = \mathbf{795.5\ Hz.}$$

## 9.8.2 Constant – K High Pass Filters

The constant –$K$ High pass filter as $T$ and $\pi$-network are as shown in Fig. 9.21 ($a$) and ($b$) respectively. This type of filter can be obtained by changing the positions of series and shunt elements of the constant –$K$ low pass filter.

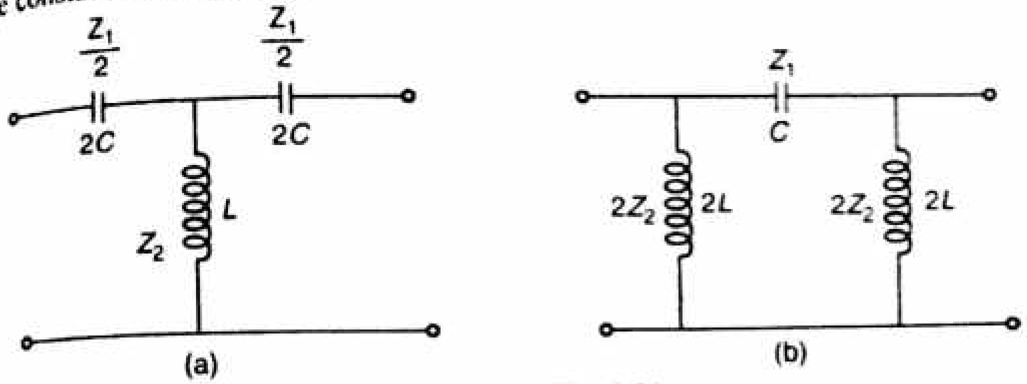



Fig. 9.21

Let $Z_1 = \dfrac{1}{j\omega C}$ and $Z_2 = j\omega L$, then design impedance for the load impedance is given by

$$R_0^2 = Z_1 Z_2 = \frac{L}{C}$$

or
$$\boxed{R_0 = \sqrt{\frac{L}{C}}}$$

(i) **Cut-off Frequency:**

The cut-off frequencies are given by

$$Z_1 = 0 \qquad \text{or} \qquad \frac{1}{j\omega_c C} = 0$$

or
$$\boxed{\omega_c \text{ and hence } f_c = \infty}$$

and
$$Z_1 + 4\ Z_2 = 0$$

or
$$\frac{1}{j\omega_c C} + 4.\ j\omega_c L = 0$$

or
$$\omega_c = \frac{1}{2\sqrt{LC}}$$

or
$$\boxed{f_c = \frac{1}{2\pi\sqrt{LC}}}$$

Therefore, the pass band of constant –$K$ high pass filter extends from $\dfrac{1}{4\pi\sqrt{LC}}$ to $\infty$.

The graphical representation of $Z_1$ and $Z_2$ with variation of frequency is shown in Fig. 9.22. The cut-off frequency at the intersection of the curves $Z_1$ and $-4Z_2$ is indicated as $f_c$. The filter pass all frequencies between $f_c$ and $\infty$.

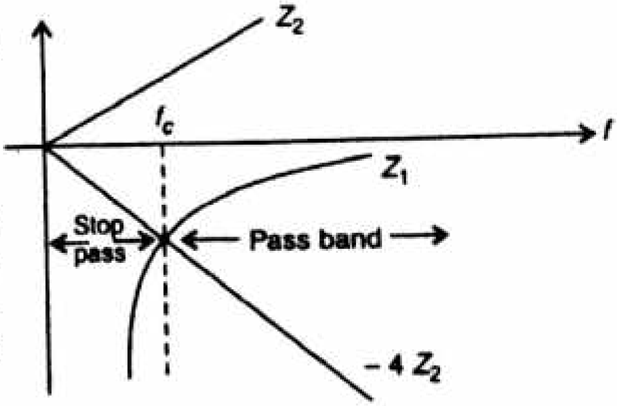



Fig. 9.22.

(*ii*) **Attenuation Constant:**

In pass-band, $\alpha = 0$

and in stop-band, $\alpha = 2 \cos h^{-1} \sqrt{\left|\frac{Z_1}{4Z_2}\right|}$

Since $Z_1 = \frac{1}{j\omega C}$ and $Z_2 = j\omega L$

$\therefore$ $\alpha = 2 \cos h^{-1} \sqrt{\frac{1}{4\omega^2 LC}}$

or $\alpha = 2 \cos h^{-1} \left(\frac{f_c}{f}\right) \left(\because f_c = \frac{1}{4\pi\sqrt{LC}}\right)$

(*iii*) **Phase Constant:**

In stop-band, $\beta = \pi$

and in pass-band, $\beta = 2 \sin^{-1} \sqrt{\left|\frac{Z_1}{4Z_2}\right|}$

$$= 2 \sin^{-1} \sqrt{\frac{1}{\omega^2 LC}}$$

$$= 2 \sin^{-1}\left(\frac{f_c}{f}\right)$$

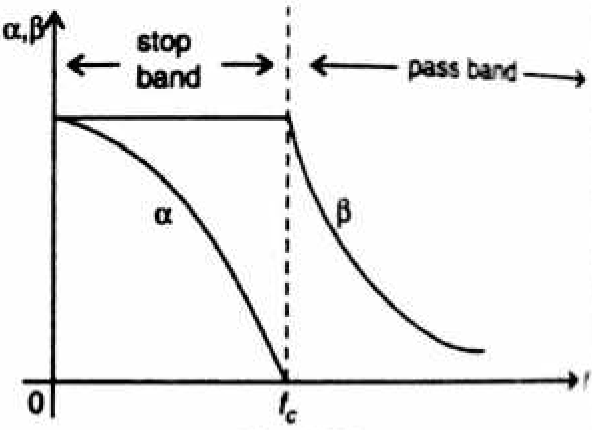



Fig. 9.23.

The variation of attenuation constant α and phase constant β with frequency $f$ is shown in Fig. 9.23.

(*iv*) **Characteristic Impedance:**

The characteristic impedance for *T*-network is given by

$$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

$\therefore$ $Z_1 = \frac{1}{j\omega C}$ and $Z_2 = j\omega L$

$\therefore$ $$Z_{0T} = \sqrt{\frac{L}{C}\left(1 - \frac{1}{4\omega^2 LC}\right)}$$

or $$\boxed{Z_{0T} = R_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

In the pass band, $f > f_c$, so that $Z_{0T}$ is real.

In the stop band, $f < f_c$, so that $Z_{0T}$ is imaginary, and if $f = f_c$, then $Z_{0T} = 0$

The characteristic impedance for π-network is given by

$$Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

$$= \sqrt{\frac{\frac{L}{C}}{1 - \frac{1}{4\,\omega^2 LC}}}$$

or

$$\boxed{Z_{0\pi} = \frac{R_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}}$$

In the pass band, $f > f_c$, so that $Z_{0\pi}$ is real.

In the stop band, $f < f_c$, so that $Z_{0\pi}$ is imaginary, and if $f = f_c$, then $Z_{0\pi} = \infty$

The variation of characteristic impedances $Z_{0T}$ and $Z_{0\pi}$ with frequency $f$ is shown in Fig. 9.24.

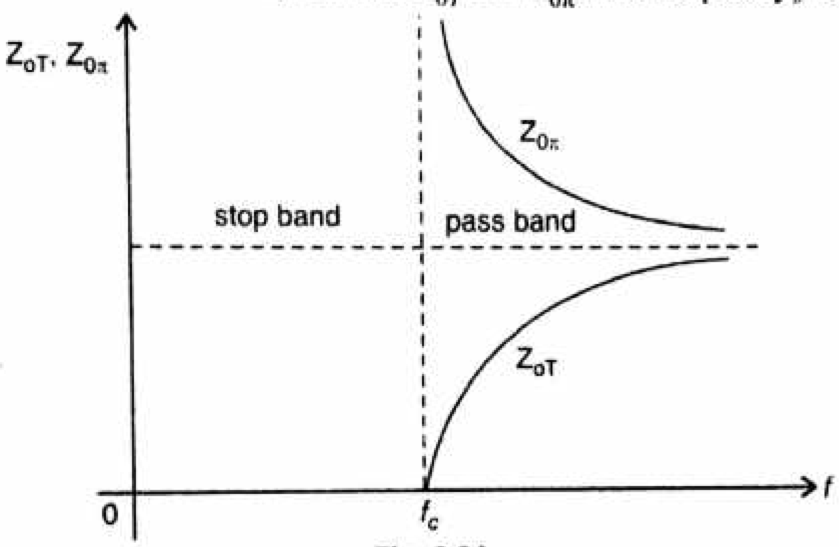



Fig. 9.24

(v) **Filter component values:**

We have, $$Z_1 Z_2 = R_0^2 = \frac{L}{C} \quad \text{... (1)}$$

and $$f_c = \frac{1}{4\pi\, LC} \quad \text{... (2)}$$

From equations (1) and (2), we get

shunt inductance, $$L = \frac{R_0}{4\pi\, f_c}$$

and series capacitance, $$C = \frac{1}{4\,\pi\, f_c R_0}$$

**Example. 9.4.** *Design a high filter as T and π-networks having a cut-off frequency of 2000 Hz with a load resistance of 300 Ω.*

**Solution.**

**Step-I. To determine the value of $L$ and $C$.**

Given, $f_c = 2000$ Hz, $R_0 = K = 300\ \Omega$

$\because$ $$K = \sqrt{\frac{L}{C}} \text{ and } f_c = \frac{1}{4\pi\sqrt{LC}}$$

From these equations, we get

$$L = \frac{K}{4\,\pi\, f} \text{ and } C = \frac{1}{4\pi\, K\, f_c}$$

$$\therefore \qquad L = \frac{300}{4\times 3.14\times 2000} = \mathbf{11.93\ mH}$$

and
$$C = \frac{1}{4\times 3.14\times 300\times 2000} = \mathbf{0.133\ \mu F}$$

**Step II: To design a high pass filter as $T$ and $\pi$-networks.**

The $T$ and $\pi$-networks of a high pass filter are shown in Fig. E 9.4($a$) and ($b$) respectively.

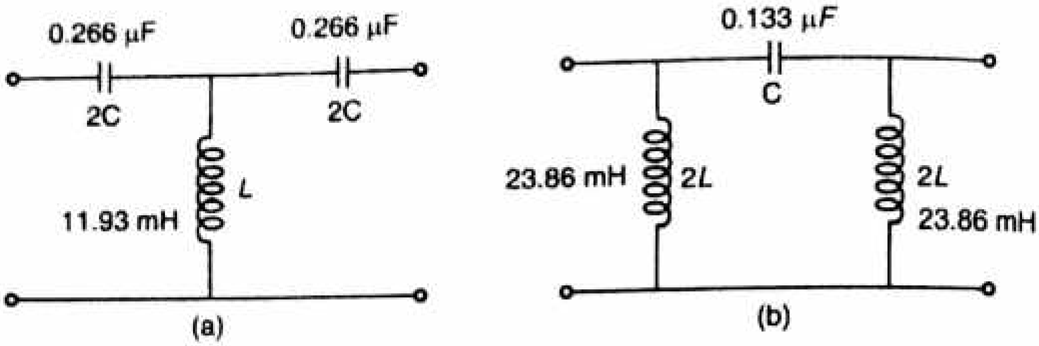



**Fig. E 9.4**

**Example. 9.5** *Determine the cut-off frequency for the high pass filter as shown in Fig. E9.5 (a) and (b).*

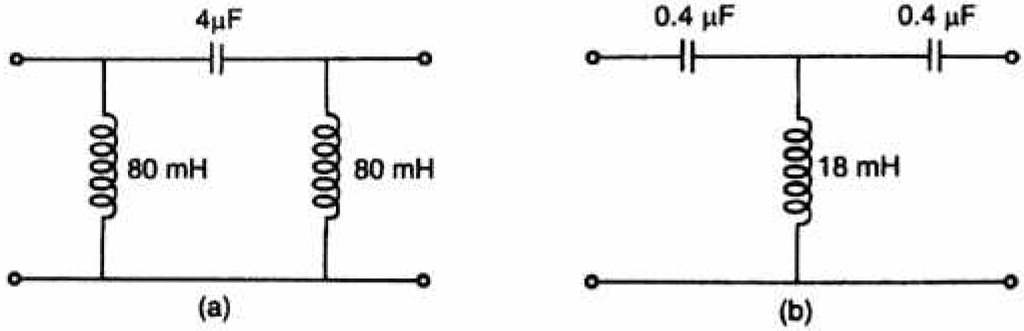



**Fig. E 9.5.**

**Solution.**

**Step I. To determine the cut-off frequency for the $\pi$-network.**

Given, $2L = 80$ mH or $L = 40$ mH

and $C = 4\ \mu F$.

We know that, the cut-off frequency for the high pass filter is given by

$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \qquad f_c = \frac{1}{2\times 3.14\times\sqrt{40\times 10^{-3}\times 4\times 10^{-6}}} = \mathbf{198.94\ Hz.}$$

**Step II. To determine the cut-off frequency for the $T$-network.**

Given, $L = 18$ mH, $2C = 0.4\ \mu F$ or $C = 0.2\ \mu F$.

$$\therefore \qquad f_c = \frac{1}{2\times 3.14\times\sqrt{18\times 10^{-3}\times 0.2\times 10^{-6}}} = \mathbf{1326\ Hz.}$$

**Note:**
$$\left[f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{K}{4\pi L} = \frac{1}{4\pi KC},\ \text{and } K = \sqrt{\frac{L}{C}}\right]$$

**Example. 9.6.** *A constant –K high pass filter having cut-off frequency of 8 KHz and nominal characteristic impedance of 600 Ω, find the component values of π-network. Hence, find its characteristic and phase constant at f = 12 KHz and attenuation at f = 0.8 kHz.*

***Solution.***

**Step I. To determine the component value of π-network.**

Given, $f_c = 8000$ Hz, $R_0 = K = 600\ \Omega$

We know that, $f_c = \dfrac{1}{4\pi\sqrt{LC}}$ amd $K = \sqrt{\dfrac{L}{C}} = R_0$

From these two equations, we get

$$L = \frac{K}{4\pi f_c} \text{ and } C = \frac{1}{4\pi K f_c}$$

$$\therefore \quad L = \frac{600}{4\times3.14\times8000} = 5.97 \text{ mH}$$

and $$C = \frac{1}{4\times3.14\times600\times8000}\ 0.0166\ \mu\text{F}$$

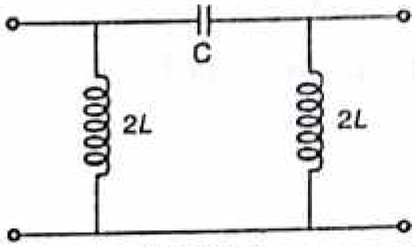



**Fig. E 9.6**

Therefore, the values of components in π-network are

$$C = \mathbf{0.0166\ \mu F}$$
$$2L = 5.97 \times 2 = \mathbf{11.94\ mH}$$

**Step II: To determine the characteristic impedance and phase constant at $f$ = 12 KHz.**

We know that at any frequency $f$, the characteristic impedance and phase constant for π-section high pass filter is given by

$$Z_{0\pi} = \frac{R_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

and $$\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right)$$

$$\therefore \quad Z_{0\pi} = \frac{600}{\sqrt{1-\left(\frac{8000}{12000}\right)^2}} = \mathbf{805\ \Omega}$$

and $$\beta = 2\sin^{-1}\left(\frac{8000}{12000}\right) = \mathbf{83.6^\circ = 1.46\ rad.}$$

**Step III: To determine attenuation at $f$ = 0.8 KHz.**

We know that, attenuation at $f$ is given by

$$\alpha = 2\cos h^{-1}\left(\frac{f_c}{f}\right)$$

$$\therefore \quad \alpha = 2\cos h^{-1}\left(\frac{8000}{800}\right) = \mathbf{5.99\ nepers.}$$

**Example 9.7.** *Design a prototype high pass filter T and π sections if characteristic impedance is 500 Ω and cut-off frequency of 2000 Hz.*

**Solution.**

**Step I. To determine the component values of proto type high pass filter.**

Given, $R_0 = 500\ \Omega$, $f_c = 2000$ Hz.

We know that by prototype high pass filters,

$$f_c = \frac{1}{4\pi\sqrt{LC}} \text{ and } K = R_0 = \sqrt{\frac{L}{C}}$$

From these equations, we get

$$L = \frac{R_0}{4\pi f_c} \text{ and } C = \frac{1}{4\pi R_0 f_c}$$

$$\therefore \qquad L = \frac{5000}{4\times 3.14\times 2000} = \mathbf{19.89\ mH},$$

and

$$C = \frac{1}{4\times 3.14\times 500\times 2000} = \mathbf{79.57\ nF}$$

**Step II: To obtain T-network of proto type high pass filter.**

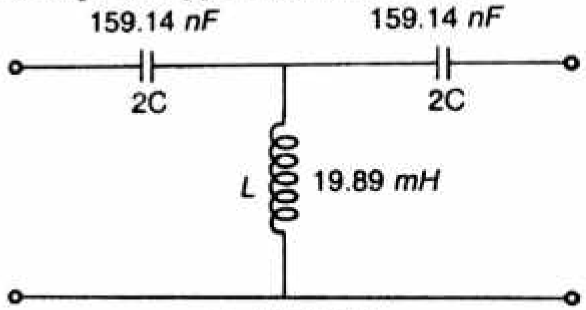



Fig. E 9.7 (a)

**Step III: To obtain π-network of proto type high pass filter.**

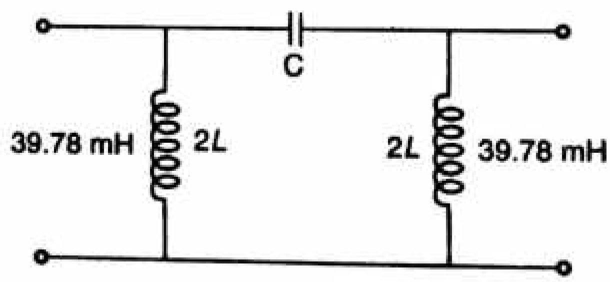



Fig. E 9.7 (b)

### 9.8.3 Constant-K Band Pass Filters

The constant-$K$ band pass filter as $T$ and π-networks are as shown in Fig. 9.25 (a) and (b) respectively. It is a srires combination of a low pass filter and a high pass filter with $f_{c_2}$ cut-off frequency of the low pass filter and $f_{c_1}$ cut-off frequency of the high pass filter. Where $f_{c_1} < f_{c_2}$. Thus the overlap allow only a band of frequencies to pass.

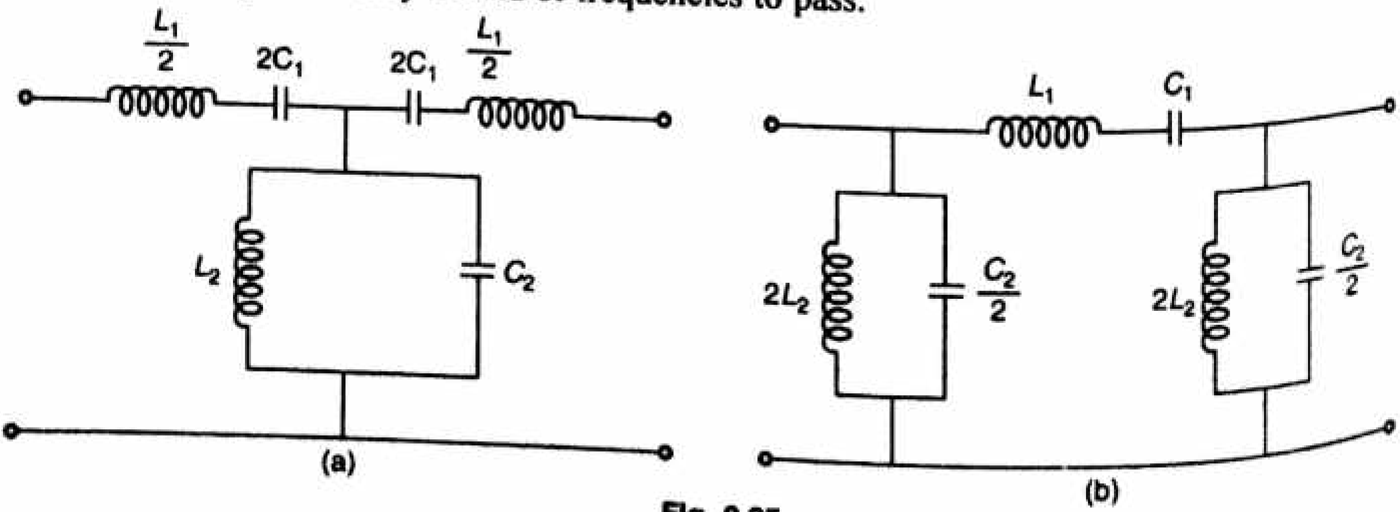



Fig. 9.25

**For T-network :**

The impedance of series arm, $\frac{Z_1}{2} = \frac{j\omega L_1}{2} + \frac{1}{j\omega.2C_1}$

or $$Z_1 = j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)$$

The impedance of shunt arm, $Z_2 = j\omega L_1 \parallel \frac{1}{j\omega C_2}$

or $$Z_2 = \frac{j\omega L_2}{1-\omega^2 L_2 C_2}$$

Therefore, $$Z_1 Z_2 = \frac{L_2(1-\omega^2 L_1 C_1)}{C_1(1-\omega^2 L_2 C_2)}$$

For the series arm, the frequency of resonance is given by

$$\omega_0 = \frac{1}{\sqrt{L_1 C_1}}$$

and for the shunt arm, the frequency of resonance is given by

$$\omega_0 = \frac{1}{\sqrt{L_2 C_2}}$$

when both resonance frequencies are equal then there will be a single pass band.

Then, we get $\frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}}$

or $$L_1 C_1 = L_2 C_2$$

Therefore, $$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = R_0^2$$

or $$\boxed{R_0 = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}}}$$

**Cut-off Frequencies:**

The cut-off frequencies are given by

$$Z_1 + 4\,Z_2 = 0$$

or $$Z_1 = -4\,Z_2$$

Multiplying both sides by $Z_1$, we get

$$Z_1^2 = -4\,Z_1 Z_2 = -4\,R_0^2$$

or $$Z_1 = \pm j\,2R_0$$

This equation gives two cut-off frequencies:

At $$f_{c_1},\ Z_1 = -j\,2R_0$$

At $$f_{c_2},\ Z_1 = +j\,2R_0$$

Therefore, the impedance $Z_1$ of the series arm at $f_{c_1}$ is negative of the impedance $Z_1$ at $f_{c_2}$ i.e.

$$j\left[\omega L_1 - \frac{1}{\omega C_1}\right]_{f_{c_1}} = -j\left[\omega L_1 - \frac{1}{\omega C_1}\right]_{f_{c_2}}$$

or
$$\omega_1 L_1 - \frac{1}{\omega_1 C_1} = -\left(\omega_2 L_1 - \frac{1}{\omega_2 C_1}\right)$$

$$\omega_1^2 L_1 C_1 - 1 = -\frac{\omega_1}{\omega_2}\left(\omega_2^2 L_1 C_1 - 1\right)$$

Since
$$L_1 C_1 = \frac{1}{\omega_0^2} \text{ and } \omega_0 = 2\pi f_0$$

Therefore,
$$\left(\frac{f_{c_1}}{f_0}\right)^2 - 1 = -\frac{f_{c_1}}{f_{c_2}}\left[\left(\frac{f_{c_2}}{f_0}\right)^2 - 1\right]$$

or
$$f_0^2 - f_{c_1}^2 = -\frac{f_{c_1}}{f_{c_2}}\left(f_{c_2}^2 - f_0^2\right)$$

$$f_0^2\left(1 + \frac{f_{c_1}}{f_{c_2}}\right) = f_{c_1} f_{c_2} + f_{c_1}^2$$

or
$$f_0^2 = f_{c_1} f_{c_2}$$

Therefore,
$$\boxed{f_0 = \sqrt{f_{c_1} f_{c_2}}}$$

Hence, the resonant frequency of the band pass filter is equal to the geometric mean of the two cut-off frequencies.

**(*ii*) Filter Component values:**

The filter components are $C_1$, $L_1$, $C_2$ and $L_2$.

We have, at cut-off frequency $f_{c_1}$,

$$Z_1 = -j\,2R_0$$

or
$$\left(j\omega_1 L_1 + \frac{1}{j\omega_1 C_1}\right) = -j\,2\,R_0$$

$$\frac{\omega_1^2 L_1 C_1 - 1}{\omega_1 C_1} = -2\,R_0$$

$$\frac{\omega_1^2}{\omega_0^2} - 1 = -2\,R_0\,\omega_1\,C_1 \qquad \left(\text{since, } \omega_0^2 = \frac{1}{L_1 C_1}\right)$$

or
$$1 - \left(\frac{f_{c_1}}{f_0}\right)^2 = 4\pi R_0\, f_{c_1} C_1 \qquad (\text{Since, } \omega_1 = 2\pi f_{c_1})$$

$$1 - \frac{f_{c_1}^2}{f_{c_1} f_{c_2}} = 4\pi R_0\, f_{c_1} C_1 \qquad \left(\text{Since, } f_0 = \sqrt{f_{c_1} f_{c_2}}\right)$$

$$f_{c_2} - f_{c_1} = 4\pi R_0 f_{c_1} f_{c_2} C_1$$

or

Therefore,

$$\boxed{C_1 = \frac{f_{c_2} - f_{c_1}}{4\pi R_0 f_{c_1} f_{c_2}}}$$

Since,

$$L_1 C_1 = \frac{1}{\omega_0^2}$$

or

$$L_1 = \frac{1}{4\pi^2 f_0^2 C_1}$$

or

$$\boxed{L_1 = \frac{1}{4\pi^2 f_{c_1} f_{c_2} C_1}} \qquad \left(Since\ f_0^2 = f_{c_1} f_{c_2}\right)$$

Putting the value of $C_1$, then we get

$$\boxed{L_1 = \frac{R_0}{\pi\left(f_{c_2} - f_{c_1}\right)}}$$

We have,

$$R_0 = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}}$$

or

$$L_2 = C_1 R^2{}_0$$

Putting the value of $C_1$ then we get

$$\boxed{L_2 = \frac{\left(f_{c_2} - f_{c_1}\right) R_0}{4\pi f_{c_1} f_{c_2}}}$$

and

$$C_2 = \frac{L_1}{R_0^2}$$

Putting the value of $L_1$, then we get

$$\boxed{C_2 = \frac{1}{\pi R_0 \left(f_{c_2} - f_{c_1}\right)}}$$

**Example. 9.8.** *Design a constant – K band pass filter with cut off frequencies 3 KHz and 7.5 KHz and nominal characteristic impedance of 900 Ω.*

**Solution. Step 1: To determine the values of components of a constant –*K* band pass filter.**

Given, $f_{c_1} = 3$ kHz, $f_{c_2} = 7.5$ kHz, $R_0 = 900\ \Omega$.

We know that,

$$L_1 = \frac{R_0}{\pi\left(f_{c_2} - f_{c_1}\right)} = \frac{900}{3.14\left(7.5\times10^3 - 3\times10^3\right)} = 63.66 \text{ mH}$$

$$L_2 = \frac{R_0\left(f_{c_2} - f_{c_1}\right)}{4\pi f_{c_1} f_{c_2}} = \frac{900\left(7.5\times10^3 - 3\times10^3\right)}{4\times3.14\times3\times10^3\times7.5\times10^3} = 14.32 \text{ mH}$$

$$C_1 = \frac{(f_{c_2} - f_{c_1})}{4\pi R_0 f_{c_1} f_{c_2}} = \frac{7.5 \times 10^3 - 3 \times 10^3}{4 \times 3.14 \times 900 \times 3 \times 10^3 \times 7.5 \times 10^3} = 0.017\ \mu F$$

$$C_2 = \frac{1}{\pi R_0 (f_{c_2} - f_{c_1})} = \frac{1}{3.14 \times 900 (7.5 \times 10^3 - 3 \times 10^3)} = 0.078\ \mu F$$

Therefore, $\frac{L_1}{2} = \frac{63.66}{2} = 31.83$ mH and $2C_1 = 2 \times 0.017\ \mu F = 0.034\ \mu F$

**Step II: To obtain constant $-k$ band pass filter circuit.** The constant $-k$ band pass filter is shown in Fig. E 9.8.

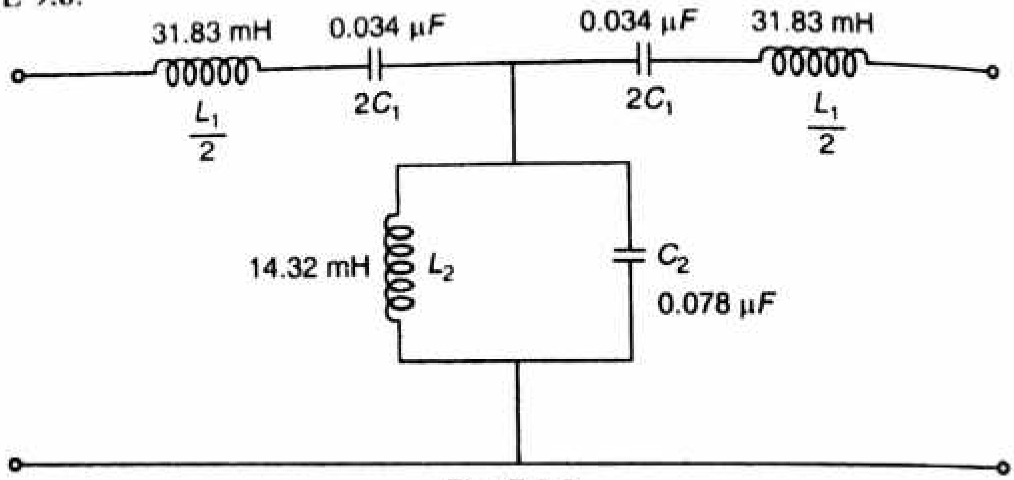



**Fig. E 9.8**

**Example 9.9.** *Design a proto type band pass filter with cut-off frequencies 1.25 kHz and 2 kHz and nominal characteristic impedance of 4kΩ.*

**Solution.**

**Step I: To determine the value of components of a proto type band pass filter.**

Given, $f_{c_1} = 1.25$ kHz, $f_{c_2} = 2$ kHz, $R_0 = 4$ kΩ.

We know that,

$$L_1 = \frac{R_0}{\pi (f_{c_2} - f_{c_1})} = \frac{4000}{3.14(2000 - 1250)} = 1.697\ H$$

$$L_2 = \frac{R_0 (f_{c_2} - f_{c_1})}{4\pi f_{c_1} f_{c_2}} = \frac{4000\ (2000 - 1250)}{4 \times 3.14 \times 1250 \times 2000} = 0.0954\ H$$

$$C_1 = \frac{(f_{c_2} - f_{c_1})}{4\pi R_0 f_{c_1} f_{c_2}} = \frac{(2000 - 1250)}{4 \times 3.14 \times 4000 \times 1250 \times 2000} = 5.968\ nF$$

$$C_2 = \frac{1}{\pi R_0 (f_{c_2} - f_{c_1})} = \frac{1}{3.14 \times 4000(2000 - 1250)} = 106.1\ nF$$

Therefore, $\frac{L_1}{2} = \frac{1.697}{2} = 0.848$ H, $2C_1 = 2 \times 5.968$ nF $= 11.93$ nF

$2L_2 = 2 \times 0.954 = 0.19$ H and $\frac{C_2}{2} = \frac{106.1\ nF}{2} = 53.05$ nF

**Step II: To obatin prototype band pass filter as $T$ and $\pi$-networks.**

The prototype band pass filter as $T$ and $\pi$-sections are as shown in Fig. E 9.9 ($a$) and ($b$) respectively.

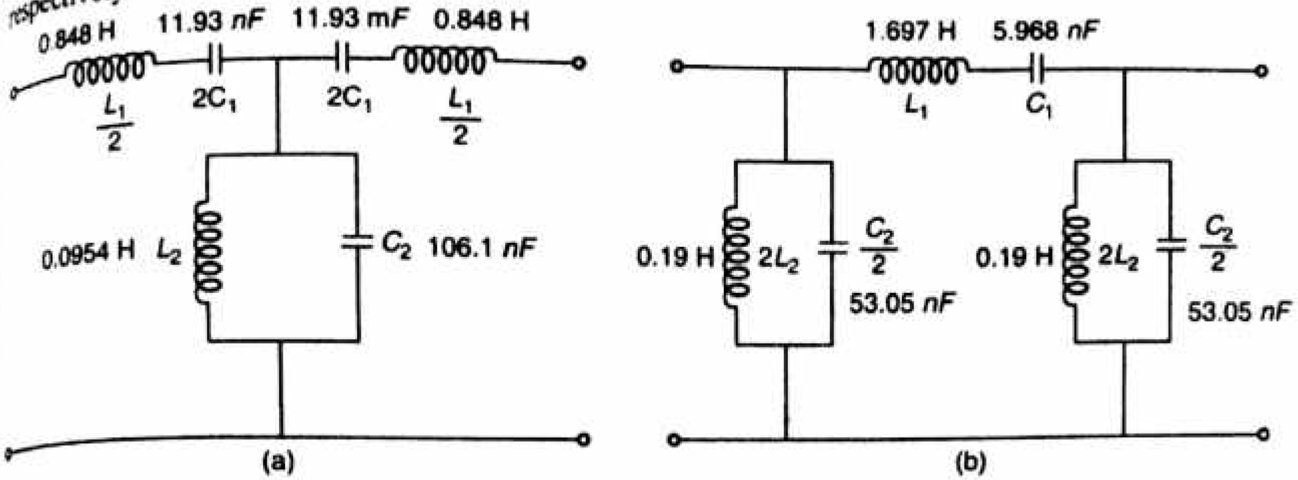



**Fig. E 9.9.**

### 9.8.4 Constant-K Band Stop Filters

The constant-$K$ band stop filter as $T$ and $\pi$-network are as shown in Fig. 9.26 ($a$) and ($b$) respectively. It is a parallel combination of a low pass filter and a high pass filter with $f_{c1}$ cut-off frequency of the low pass filter and $f_{c_2}$ cut-off frequency of the high pass filter. Where $f_{c_1} < f_{c_2}$.

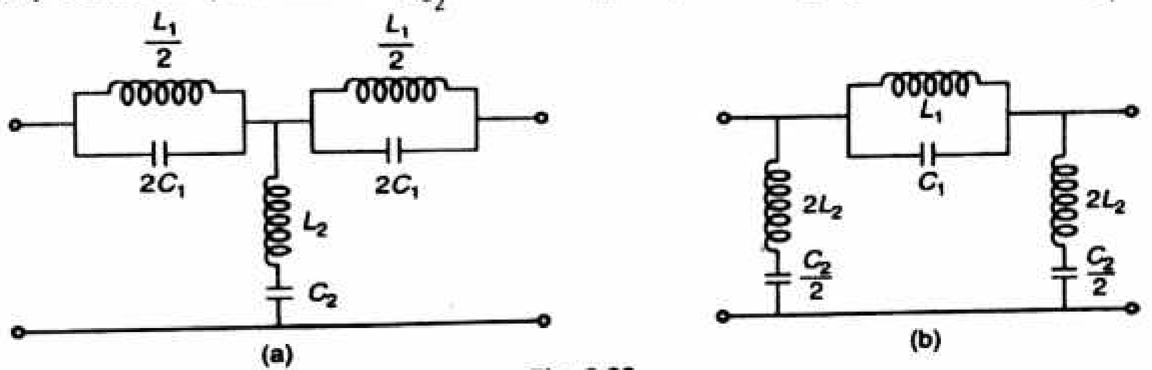



**Fig. 9.26.**

**For T-network:**

The impedance of series arm, $\dfrac{Z_1}{2} = \dfrac{j\omega L_1}{2} \parallel \dfrac{1}{j\omega.2C_1}$

or
$$\frac{Z_1}{2} = \frac{\left(\dfrac{j\omega L_1}{2}\right).\left(\dfrac{1}{j2\omega C_1}\right)}{\dfrac{j\omega L_1}{2} + \dfrac{1}{j2\omega C_1}} = \frac{j\omega L_1}{2(1-\omega^2 L_1 C_1)}$$

or
$$Z_1 = \frac{j\omega L_1}{1-\omega^2 L_1 C_1}$$

The impedance of shunt arm, $Z_2 = j\omega L_2 + \dfrac{1}{j\omega C_2}$

or
$$Z_2 = \frac{-\omega^2 L_2 C_2 + 1}{j\omega C_2}$$

or
$$Z_2 = j\left(\frac{\omega^2 L_2 C_2 - 1}{\omega C_2}\right)$$

Therefore, $$Z_1 Z_2 = \frac{L_1(1-\omega^2 L_2 C_2)}{C_2(1-\omega^2 L_1 C_1)}$$

For the series arm, the resonant frequency is given by

$$\omega_0 = \frac{1}{\sqrt{L_1 C_1}}$$

And for the shunt arm, the resonant frequency is given by $\omega_0 = \frac{1}{\sqrt{L_2 C_2}}$

For equal resonant frequency, we get

$$\frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}}$$

or $$L_1 C_1 = L_2 C_2$$

Therefore, $$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = R_0^2$$

or $$\boxed{R_0 = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}}}$$

**(i) Cut-off Frequencies**

The cut-off frequencies are given by

$$Z_1 + 4 Z_2 = 0$$

or $$Z_1 = -4 Z_2$$

Multiplying both sides by $Z_1$, we get

$$Z_1^2 = -4 Z_1 Z_2 = -4 R_0^2$$

or $$Z_1 = \pm j2 R_0$$

This equation gives two cut-off frequencies.

At $f_{c1}$, $$Z_1 = +j2 R_0$$

At $f_{c2}$, $$Z_1 = j2 R_0$$

Therefore, the impedance $Z_1$ of the series arm at $f_{c1}$ is negative of the impedance at $f_{c2}$. *i.e.,*

$$\left[\frac{j\omega L_1}{1-\omega^2 L_1 C_1}\right]_{f_{c_1}} = -\left[\frac{j\omega L_1}{1-\omega^2 L_1 C_1}\right]_{f_{c_2}}$$

or $$j\,\omega_1 L_1 (1-\omega_2^2 L_1 C_1) = -j\,\omega_2 L_1 (1-\omega_1^2 L_1 C_1)$$

$$1-\omega_1^2 L_1 C_1 = \frac{\omega_1}{\omega_2}\left(\omega_2^2 L_1 C_1 - 1\right)$$

Since, $$L_1 C_1 = \frac{1}{\omega_0^2} \quad \text{and} \quad \omega_0 = 2\pi f_0$$

Therefore, $$1-\left(\frac{\omega_1}{\omega_0}\right)^2 = \frac{\omega_1}{\omega_2}\left[\left(\frac{\omega_2}{\omega_0}\right)^2 - 1\right]$$

or $$1-\left(\frac{f_{c_1}}{f_0}\right)^2 = \frac{f_{c_1}}{f_{c_2}}\left[\left(\frac{f_{c_2}}{f_0}\right)^2 - 1\right]$$

or $$f_0^2 - f_{c_1}^2 = \frac{f_{c_1}}{f_{c_2}}\left(f_{c_2}^2 - f_0^2\right)$$

$$f_0^2\left(1+\frac{f_{c_1}}{f_{c_2}}\right) = f_{c_1} f_{c_2} + f_{c_1}^2$$

or $$f_0^2 = f_{c1} f_{c2}$$

Therefore, $$\boxed{f_0 = \sqrt{f_{c_1} f_{c_2}}}$$

Hence, the resonant frequency of the band stop filter is also equal to the geometric mean of the two cut-off frequencies.

(ii) **Filter Component Values**

The filter components are $C_1$, $L_1$, $C_2$ and $L_2$.

We have, at cut-off frequency $f_{c_1}$,

$$Z_1 = +j2\,R_0$$

or $$\frac{j\omega_1 L_1}{1-\omega^2 L_1 C_1} = j2\,R_0$$

or $$\frac{\omega_1 L_1}{1-\left(\frac{\omega_1}{\omega_0}\right)^2} = 2\,R_0 \qquad \left(\text{Since, } \omega_0^2 = \frac{1}{L_1 C_1}\right)$$

$$1-\left(\frac{f_{c_1}}{f_0}\right)^2 = \frac{\omega_1 L_1}{2R_0} \qquad (\text{Since, } \omega_1 = 2\pi f_1)$$

$$1-\frac{f_{c_1}}{f_{c_1} f_{c_2}} = \frac{\omega_1 L_1}{2R_0} \qquad \left(\text{Since, } f_0 = \sqrt{f_{c_1} f_{c_2}}\right)$$

or $$f_{c_2} - f_{c_1} = \frac{\omega_1 L_1 f_{c_2}}{2R_0}$$

$$= \frac{2\pi\, f_{c_1} L_1 f_{c_2}}{2R_0} \qquad (\because\ \omega_1 = 2\pi f_{c_1})$$

Therefore, $$\boxed{L_1 = \frac{R_0(f_{c_2} - f_{c_1})}{\pi\, f_{c_1} f_{c_2}}}$$

Since, $$L_1\,C_1 = \frac{1}{\omega_0^2}$$

or $$C_1 = \frac{1}{4\pi^2 f_0^2 L_1}$$

Putting the value of $L_1$, then we get

$$\boxed{C_1 = \frac{1}{4\pi R_0(f_{c_2} - f_{c_1})}}$$

We have $$R_0 = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}}$$

or $$L_2 = R_0^2\, C_1$$

Putting the value of $C_1$, then we get

$$\boxed{L_2 = \frac{R_0}{4\pi\,(f_{c_2} - f_{c_1})}}$$

and $$C_2 = \frac{L_1}{R_0^2}$$

Putting the value of $L_1$, then we get

$$\boxed{C_2 = \frac{f_{c_2} - f_{c_1}}{\pi\, R_0\, f_{c_1}\, f_{c_2}}}$$

**Example 9.10.** *Design a constant-K band stop filter with cut-off frequencies of 3 kHz and 7.5 kHz and nominal characteristic impedance of 900 Ω.*

**Solution.**

**Step-I. To determine the value of components of a constant-K band stop filter.**

Given, $f_{c1}$ = 3 kHz, $f_{c2}$ = 7.5 kHz, $R_0$ = 900 Ω

We know that,

$$L_1 = \frac{R_0(f_{c_2} - f_{c_1})}{\pi\, f_{c_1} f_{c_2}} = \frac{900\,(7.5\times10^3 - 3\times10^3)}{3.14\times3\times10^3\times7.5\times10^3} = 57.3 \text{ mH}$$

$$L_2 = \frac{R_0}{4\pi\left(f_{c_2} - f_{c_1}\right)} = \frac{900}{4\times3.14(7.5\times10^3 - 3\times10^3)} = 15.9 \text{ mH}$$

$$C_1 = \frac{1}{4\pi R_0\left(f_{c_2} - f_{c_1}\right)} = \frac{1}{4\times3.14\times900\,(7.5\times10^3 - 3\times10^3)} = 0.0196\ \mu\text{F}$$

$$C_2 = \frac{f_{c_2} - f_{c_1}}{\pi\, R_0\, f_{c_1} f_{c_2}} = \frac{7.5\times10^3 - 3\times10^3}{3.14\times900\times3\times10^3\times7.5\times10^3} = 0.0707\ \mu F$$

Therefore, $\frac{L_1}{2} = \frac{57.3}{2}$ = 28.6 mH and 2 $C_1$ = 2 × 0.0196 = 0.0392 µF.

**Step-II. To obtain constant-*K* band stop filter circuit.**

The constant-K band stop filter is shown in Fig. E 9.10.

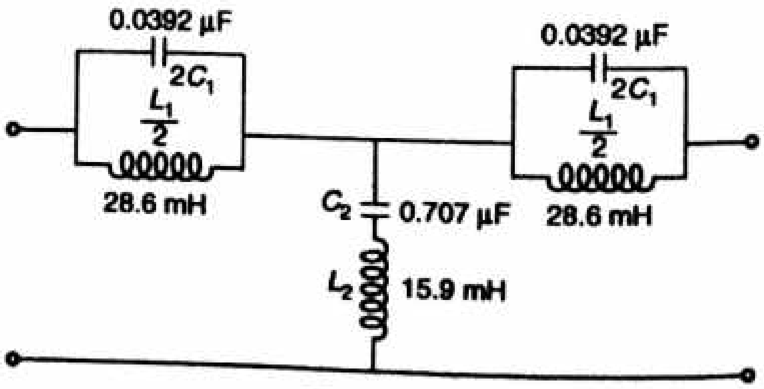



Fig. E 9.10

## 9.9 DISADVANTAGES OF CONSTANT-K TYPE FILTERS

(1) The attenuation is changing gradually in the stop band, therefore, some of the frequencies near cut-off frequency, pass through the filter.

(2) It is expected that attenuation should change very sharply in the stop band after the cut-off. But it is not changing sharply. This can be overcome by connecting two or more proto type sections of same type (either $T$ or $\pi$ sections) in series. This increases the attenuation in multiple.

(3) There is mismatch between the load and filter section. This is because characteristic impedance doesnot remain constant in the pass band.

## 9.10. *m*-DERIVED FILTERS

To overcome the disadvantages of the constant-K type filter; the characteristic impedance of each section in cascade connection must be same. All the disadvantage of constant-K type filter are overcome by designing a new filter, called as m-derived filter.

### 9.10.1. *m*-Derived T-network

Fig. 9.27 (*a*) and (*b*) shows the constant-K type T-network and corresponding m-derived filter having the same characteristic impedance respectively.

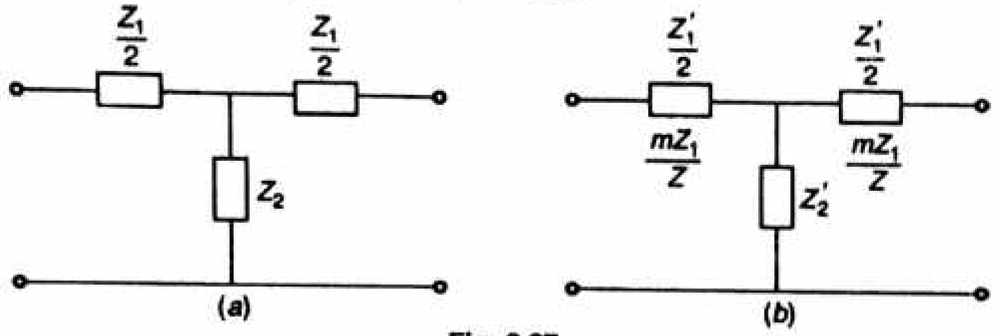



**Fig. 9.27.**

For *m*-derived T-network, we select $Z_1' = mZ_1$ and $Z_2' = Z_2$ such that characteristic impedances are equal.

The characteristic impedance of constant-K type filter is given by $Z_{oT} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}$.

The characteristic impedance for *m*-derived filter is given by

$$Z'_{oT} = \sqrt{\frac{Z'^2_1}{4} + Z'_1Z'_2} = \sqrt{\frac{m^2Z_1^2}{4} + mZ_1Z'_2}$$

Since, $\quad Z_{oT} = Z'_{oT}$

$$\therefore \quad \sqrt{Z_1Z_2 + \frac{Z_1^2}{4}} = \sqrt{\frac{m^2Z_1^2}{4} + mZ_1Z'_2}$$

Squaring both sides, then we get

$$Z_1Z_2 + \frac{Z_1^2}{3} = \frac{m^2Z_1^2}{4} + mZ_1Z'_2$$

or

$$Z_2' = \frac{Z_2}{m} + \left(\frac{1-m^2}{4m}\right)Z_1$$

It shows that the shunt arm $Z_2'$ consists of two impedances in series as shown in Fig. 9.28.

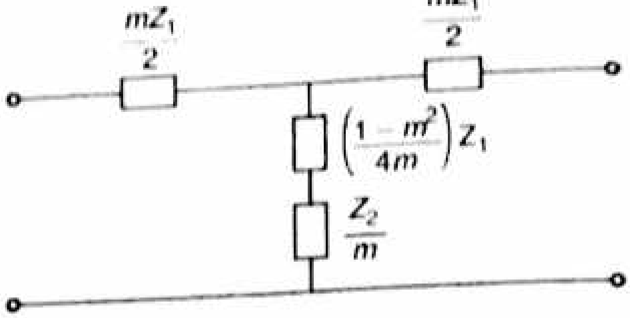



**Fig. 9.28** Complete *m*-derived T-network.

The impedances $\frac{Z_2}{m}$ and $\left(\frac{1-m^2}{4m}\right)Z_1$ can be obtain by keeping $0 < m < 1$. If $m = 1$, then a constant-$K$ type $T$-network is obtained.

### 9.10.2. *m*-Derived π-network

Fig. 9.29 (*a*) and (*b*) shows the constant $K$-type π-network and corresponding m-derived filter having the same characteristic impedance respectively.

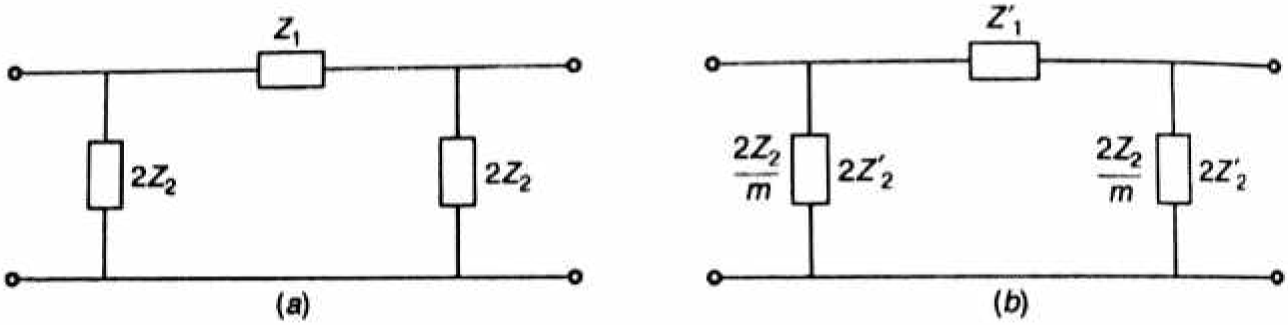



**Fig. 9.29.**

For *m*-derived π-network, we select $Z_2' = \frac{Z_2}{m}$ and $Z_1' = Z_1$ such that characteristic impedances are equal.

The characteristic impedance of constant-$K$ type π-network filter is given by

$$Z_{o\pi} = \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}}$$

The characteristic impedance of *m*-derived π-network filter is given by

$$Z'_{o\pi} = \sqrt{\frac{Z'_1\,Z'_2}{1+\frac{Z'_1}{4\,Z'_2}}} = \sqrt{\frac{Z'_1\frac{Z_2}{m}}{1+\frac{Z'_1}{\left(\frac{4Z_2}{m}\right)}}}$$

Since, $Z_{o\pi} = Z'_{o\pi}$

$$\therefore \qquad \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z'_1\frac{Z_2}{m}}{1+\frac{Z'_1}{\left(\frac{4Z_2}{m}\right)}}}$$

Squaring both sides, then we get

$$\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}} = \frac{Z'_1 Z_2}{m\left(1+\frac{mZ'_1}{4Z_2}\right)}$$

or $$mZ_1 Z_2\left(1+\frac{mZ'_1}{4Z_2}\right) = Z'_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)$$

$$Z'_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right) - \frac{m^2 Z_1}{4} Z'_1 = mZ_1 Z_2$$

or $$Z'_1 = \frac{4\,mZ_1 Z_2}{Z_1(1-m^2)+4Z_2}$$

$$= \frac{4m^2 Z_1 Z_2}{mZ_1(1-m^2)+4mZ_2}$$

or $$Z'_1 = \frac{mZ_1\left(\frac{4m}{1-m^2}\right)Z_2}{mZ_1 + \left(\frac{4m}{1-m^2}\right)Z_2}$$

It shows that the series arm $Z'_1$ consists of two impedances in parallel as shown in Fig. 9.30.

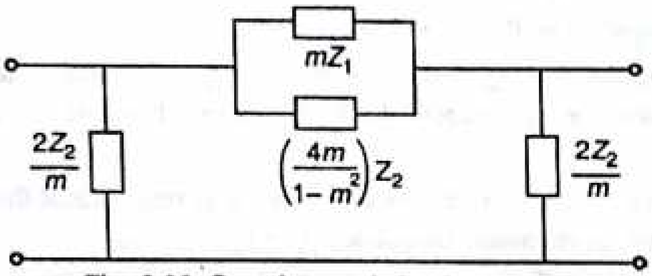



Fig. 9.30. Complete $m$-derived $\pi$-network

The impedances $mZ_1$ and $\left(\frac{4m}{1-m^2}\right)Z_2$ can be obtained by keeping $0 < m < 1$. If $m = 1$, then a constant-$K$ type $\pi$-network is obtained.

### 9.10.3. *m*-Derived Low Pass Filter

Fig. 9.31 (*a*) and (*b*) shows the $m$-derived $T$ and $\pi$-networks of low pass filter respectively.

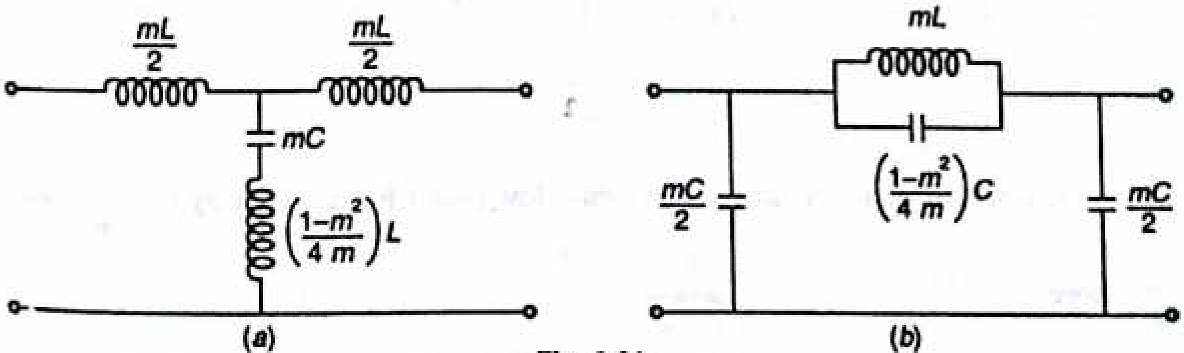



Fig. 9.31.

For low pass filter, we have

$$Z_1 = j\omega L \text{ and } Z_2 = \frac{1}{j\omega C}$$

**For *T*-network,** Shunt arm is to be chosen so that it is resonant at some frequency $f_\infty$ above cut-off frequency $f_c$. Therefore, at resonant frequency $f_r = f_\infty$.

We have, $$\left|\frac{Z_2}{m}\right| = \left|\left(\frac{1-m^2}{4m}\right)Z_1\right|$$

Putting the values of $Z_1$ and $Z_2$, we get

$$\frac{1}{m\,\omega_r C} = \left(\frac{1-m^2}{4m}\right)\omega_r L$$

$$\omega_r^2 = \frac{4}{(1-m^2)LC} = 4\pi^2 f_r^2 \qquad (\because \omega_r = 2\pi f_r)$$

or $$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}} = f_\infty$$

Since, the cut-off frequency of the constant-*K* low pass filter is given by $f_c = \dfrac{1}{\pi\sqrt{LC}}$.

Therefore, $$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$

or $$\boxed{m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}}$$

Since $f_\infty > f_c$, we must have $0 < m < 1$.

If a sharp cut-off is desired, $f_\infty$ should be near to $f_c$. If $f_c$ and $f_\infty$ are known then $m$ can be obtained. Thus, all the component values of the *m*-derived low pass filter of T-network may be obtained.

**For π-network,** series arm is to be chosen so that it is resonant at the frequency $f_\infty$ above cut-off frequency $f_c$. Therefore, at resonant frequency $f_r = f_\infty$.

We have, $$|mZ_1| = \left|\left(\frac{4m}{1-m^2}\right)Z_2\right|$$

Putting the values of $Z_1$ and $Z_2$, we get

$$m\,\omega_r L = \frac{1}{\omega_r C}\cdot\left(\frac{4m}{1-m^2}\right)$$

$$\omega_r^2 = \frac{4}{LC\,(1-m^2)} = 4\pi^2 f_r^2$$

or $$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}} = f_\infty$$

Since, the cut-off frequency for the constant-*k* low pass filter is given by $f_c = \dfrac{1}{\pi\sqrt{LC}}$.

Therefore, $$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$

or $$\boxed{m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}}$$

This is the same as that of the *T*-network low pass filter.

**Example. 9.11.** *Design m-derived T and π-networks low pass filter having nominal characteristic impedance $R_0 = 900\ \Omega$, cut-off frequency $f_c = 0.9$ kHz and infinite attenuation (or resonant) frequency $f_\infty = 1000$ Hz.*

**Solution.**

**Step-I: To determine the values of *m*, *L* and *C* of an *m*-derived low pass filter.**

Given $R_0 = 900\ \Omega, f_c = 0.9\text{ kHz} = 900\text{ Hz}, f_\infty = 1000\text{ Hz}.$

For low pass filter, we have

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}$$

$$m = \sqrt{1-\left(\frac{900}{1000}\right)^2} = 0.436$$

$$L = \frac{R_0}{\pi f_c} = \frac{900}{3.14\times 900} = 318.3\text{ mH} \qquad (\because R_0 = K)$$

and
$$C = \frac{1}{\pi R_0 f_c} = \frac{1}{3.14\times 900\times 900} = 0.393\ \mu\text{F}$$

**Step-II: To determine the value of components of *T*-networks of m-derive low pass filter.**

$$\frac{mL}{2} = \frac{0.436\times 318.3\times 10^{-3}}{2} = 69.4\text{ mH}$$

$$mC = 0.436\times 0.393\times 10^{-6} = 0.171\ \mu\text{F}$$

The m-derived *T*-network low pass filter is shown in Fig. *E* 9.11 (*a*)

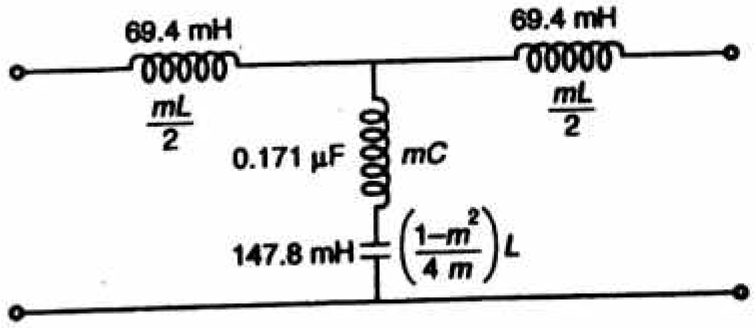



**Fig. E 9.11 (*a*).**

**Step-III:** To determine the value of components of π-network of m-derived low pass filter.

$$mL = 0.436\times 318.3\times 10^{-3} = 138.8\text{ mH}$$

$$\frac{mC}{2} = \frac{0.436\times 0.393\times 10^{-6}}{2} = 0.0855\ \mu\text{F}$$

$$\left(\frac{1-m^2}{4m}\right)C = \frac{[1-(0.436)^2]\times 0.393\times 10^{-6}}{4\times 0.436} = 0.182\ \mu\text{F}$$

The *m*-derived π-network low pass filter is shown in Fig. E 9.11 (*b*)

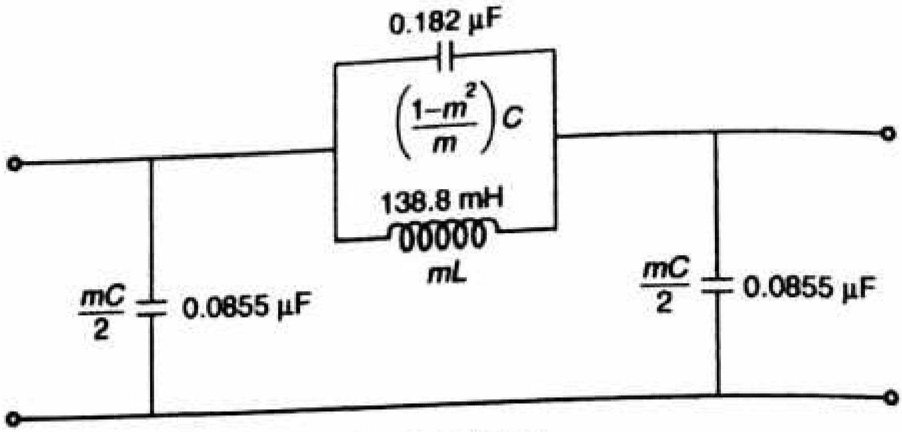



**Fig. E 9.11 (b)**

### 9.10.4. *m*-Derived High Pass Filter

Fig. 9.32 (*a*) and (*b*) shows the *m*-derived *T* and π-networks of high pass filter respectively.

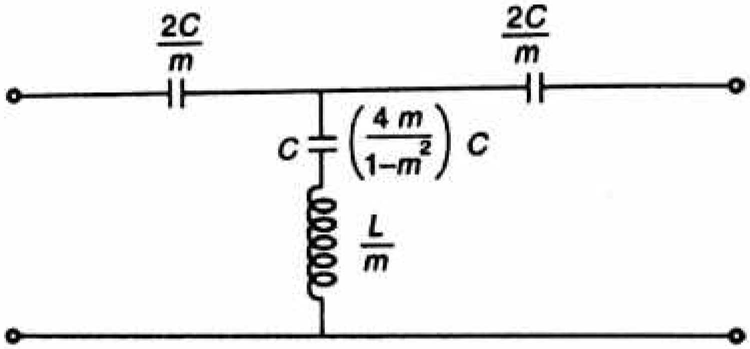



**Fig. 9.32 (a)**

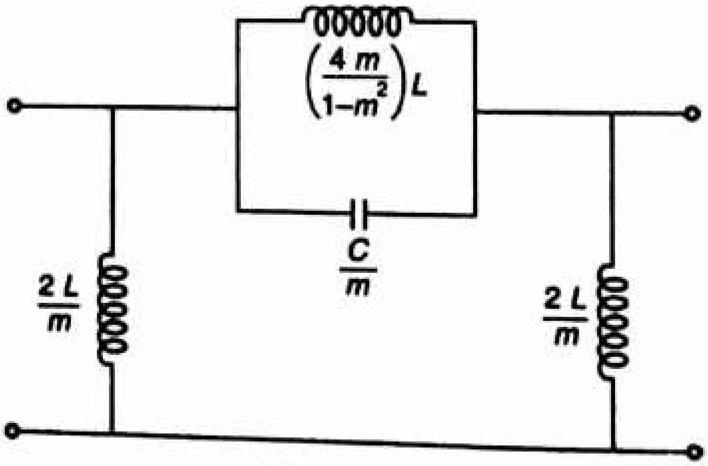



**Fig. 9.32 (b)**

For high pass filter, we have

$$Z_1 = \frac{1}{j\omega C} \text{ and } Z_2 = j\omega L$$

**For *T*-network,** shunt arm is to be chosen so that it is resonant at some frequency $f_\infty$ (resonant frequency). Therefore, at resonant frequency $f_r = f_\infty$.

We have,
$$\left|\frac{Z_2}{m}\right| = \left|\left(\frac{1-m^2}{4m}\right)Z_1\right|$$

Putting the values of $Z_1$ and $Z_2$, we get

$$\frac{\omega_r L}{m} = \left(\frac{1-m^2}{4m}\right)\cdot\frac{1}{\omega_r C}$$

$$\omega_r^2 = \frac{(1-m^2)}{4LC} = 4\pi^2 f_r^2$$

or
$$f_r = \frac{1}{4\pi}\sqrt{\frac{1-m^2}{LC}} = f_\infty$$

Since, the cut-off frequency for the constant-$K$ high pass filter is $f_c = \frac{1}{4\pi\sqrt{LC}}$.

Therefore,
$$f_\infty = \left(\sqrt{1-m^2}\right).f_c$$

or
$$\boxed{m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}}$$

Since $f_\infty < f_c$, we must have $0 < m < 1$. For known values of $f_\infty$ and $f_c$, $m$ can be obtained. Thus, all the component values of the $m$-derived high pass filter of $T$-network may be obtained.

**For π-network,** series arm is to be chosen so that it is resonant at the frequency $f_\infty$. Therefore, at resonant frequency $f_r = f_\infty$.

We have,
$$|mZ_1| = \left|\left(\frac{4m}{1-m^2}\right)Z_2\right|$$

$$\frac{m}{\omega_r C} = \left(\frac{4m}{1-m^2}\right)\omega_r L$$

or
$$\omega_r^2 = \frac{(1-m)^2}{4LC} = 4\pi^2 f_r^2$$

or
$$f_r = \frac{1}{4\pi}\sqrt{\frac{1-m^2}{LC}} = f_\infty$$

Since, the cut-off frequency of the constant-$K$ high pass filter is given by $f_c = \frac{1}{\pi\sqrt{LC}}$.

Therefore,
$$f_\infty = \left(\sqrt{1-m^2}\right)\cdot f_c$$

or
$$\boxed{m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}}$$

This is the same as that of the $T$-network high pass filter.

**Example 9.12.** *Design m-derived T and π-networks high pass filter having nominal characteristic impedance $R_0$ = 900 Ω, cut-off frequency $f_c$ = 2000 Hz and infinite attenuation (or resonant) frequency $f_\infty$ = 1800 HZ.*

**Solution.**

**Step-I: To determine the value of m, L and C of an m-derived high pass filter.**

Given, $R_o = 900\ \Omega, f_c = 2000$ Hz, $f_\infty = 1800$ Hz.

For high pass filter, we have

$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}$$

$$\therefore \qquad m = \sqrt{1-\left(\frac{1800}{2000}\right)^2} = 0.436$$

$$L = \frac{R_o}{4\pi f_c} = \frac{900}{4\times 3.14\times 2000} = 35.81 \text{ mH}$$

and
$$C = \frac{1}{4\pi f_c R_o} = \frac{1}{4\times 3.14\times 2000\times 900} = 0.0442\ \mu\text{F}$$

**Step-II: To determine the value of components of T-network of m-derived high pass filter.**

$$\frac{2C}{m} = \frac{2\times 0.0442\times 10^{-6}}{0.436} = 0.0202\ \mu\text{F}$$

$$\frac{L}{m} = \frac{35.81\times 10^{-3}}{0.436} = 82.13 \text{ mH}$$

$$\left(\frac{4m}{1-m^2}\right)C = \frac{4\times 0.436\times 0.0442\times 10^{-6}}{\left[1-(0.436)^2\right]} = 0.095\ \mu\text{F}$$

The m-derived T-network high pass filter is shown in Fig. E 9.12 (*a*).

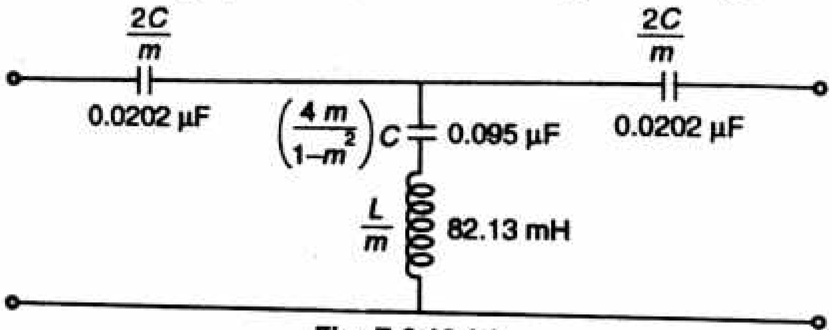



**Fig. E 9.12 (*a*)**

**Step-III: To determine the value of components of $\pi$-network of m-derived high pass filter.**

$$\frac{2L}{m} = \frac{2\times 35.81\times 10^{-3}}{0.436} = 164.26 \text{ mH}$$

$$\frac{C}{m} = \frac{0.0442\times 10^{-6}}{0.436} = 0.101\ \mu\text{F}$$

$$\left(\frac{4m}{1-m^2}\right)L = \frac{4\times 0.436\times 35.81\times 10^{-3}}{\left[1-(0.436)^2\right]} = 77.11 \text{ mH.}$$

The m-derived $\pi$-network high pass filter is shown in Fig. E 9.12 (*b*).

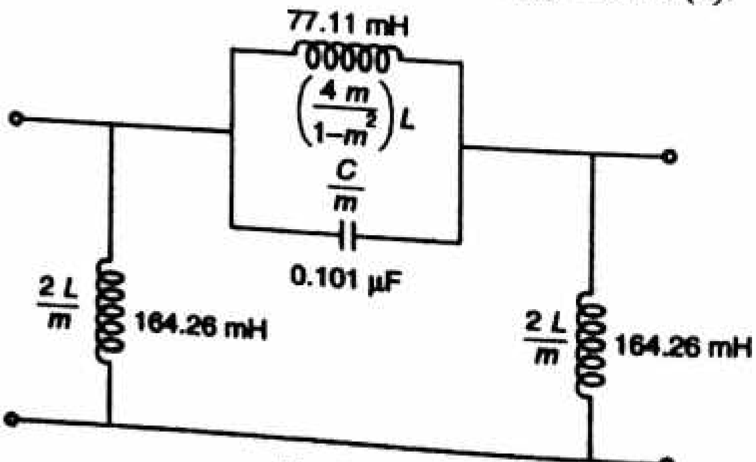



**Fig. E 9.12 (*b*)**

### [illegible] m-Derived Band Pass Filters

m-derived band pass filter is shown in Fig. 9.33.

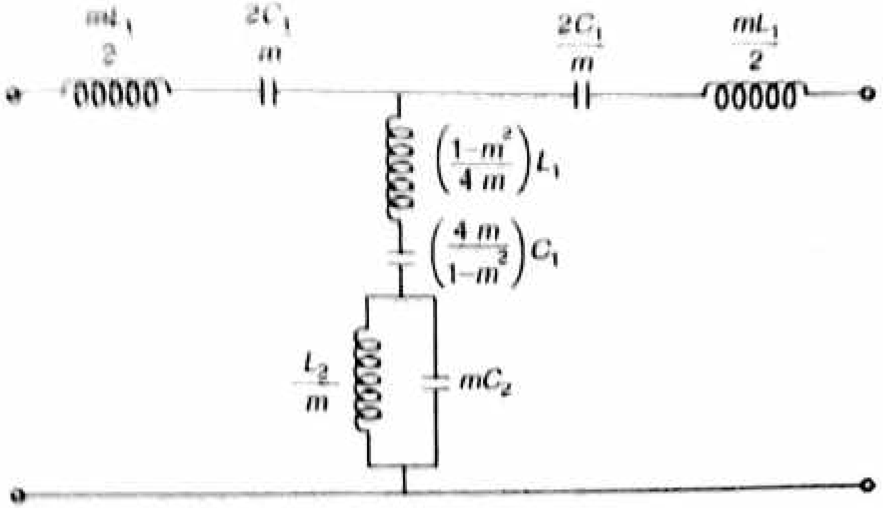



**Fig. 9.33.** *m-derived band-pass filter.*

We now derive expression for m in terms of $f_\infty$ and two cut-off frequencies $f_{c_1}, f_{c_2}$.

Infinite attenuation in the network will result when the shunt arm impedance is zero at $f_\infty$ *i.e.*, when the sum of impedance of $\left(\frac{1-m^2}{4m}\right)L_1$ and $\left(\frac{4m}{1-m^2}\right)C_1$ and impedance of parallel combination of $mC_2$ and $\frac{L_2}{m}$ becomes zero.

Therefore,

$$\left[j\omega_\infty\left(\frac{1-m^2}{4m}\right)L_1 + \frac{(1-m^2)}{j\omega_\infty 4m\,C_1}\right] + \frac{\frac{j\omega_\infty L_2}{m}\cdot\frac{1}{j\omega_\infty m\,C_2}}{\frac{j\omega_\infty L_2}{m} + \frac{1}{j\omega_\infty mC_2}} = 0$$

$$\frac{-\omega_\infty^2(1-m^2)L_1C_1 + (1-m^2)}{j\omega_\infty 4mC_1} + \frac{j\omega_\infty L_2}{-\omega_\infty^2 mL_2C_2 + m} = 0$$

$$\left[-\omega_\infty^2(1-m^2)L_1C_1 + (1-m^2)\right]\times\left[\omega_\infty^2 L_2C_2 - 1\right] + \omega_\infty^2 4L_2C_1 = 0$$

$$\left(\frac{1-m^2}{4}\right)\left(\omega_\infty^2 L_1C_1 - 1\right)\left[\omega_\infty^2 L_2C_2 - 1\right] = \omega_\infty^2 L_2C_1 \quad ...(1)$$

Since, for band pass filter

$$\left.\begin{aligned} L_1C_1 &= L_2C_2 = \frac{1}{\omega_0^2} \\ \text{and} \quad \omega_\infty &= 2\pi f_\infty,\ \omega_o = 2\pi f_o \end{aligned}\right\} \quad ...(2)$$

Then from equations (1) and (2), we get

$$\left(\frac{1-m^2}{4}\right)\left[\left(\frac{f_\infty}{f_o}\right)^2 - 1\right]^2 = 4\pi^2 f_\infty^2 L_2C_1 \quad ...(3)$$

Again for band pass filter, we have

$$f_o^2 = f_{c_1} f_{c_2};\ C_1 = \frac{f_{c_2} - f_{c_1}}{4\pi R_o f_{c_1} f_{c_2}} \text{ and } L_2 = \frac{(f_{c_2} - f_{c_1}) R_0}{4\pi f_{c_1} f_{c_2}}$$

Putting these values in equation (3), we get

$$\left(\frac{1-m^2}{4}\right)\left[\frac{f_\infty^2 - f_{c_1} f_{c_2}}{f_{c_1} f_{c_2}}\right]^2 = 4\pi^2 f_\infty^2 \left[\frac{(f_{c_2} - f_{c_1})^2}{16\pi^2 f_{c_1}^2 f_{c_2}^2}\right]$$

$$(1-m^2)\left(f_\infty^2 - f_{c_1} f_{c_2}\right)^2 = f_\infty^2 \left(f_{c_2} - f_{c_1}\right)^2$$

or $f_\infty^2 - \dfrac{f_{c_2} - f_{c_1}}{\sqrt{1-m^2}} \cdot f_\infty - f_{c_1} f_{c_2} = 0$

Therefore,

$$f_\infty = \frac{\dfrac{f_{c_2} - f_{c_1}}{\sqrt{1-m^2}} \pm \sqrt{\dfrac{(f_{c_2} - f_{c_1})^2}{(1-m^2)} + 4 f_{c_1} f_{c_2}}}{2}$$

or

$$f_\infty = \frac{f_{c_2} - f_{c_1}}{2\sqrt{1-m^2}} \pm \sqrt{\frac{(f_{c_2} - f_{c_1})^2}{4(1-m^2)} + f_{c_1} f_{c_2}} \qquad \text{...(4)}$$

From equation (4), it is clear than second term is greater than first term. Therefore, one root will be appear as a negative frequency that has no physical meaning, therefore, the expression of $f_\infty$ should be reversed. Hence, two frequencies of peak attenuation are given by

$$f_{\infty_1} = \sqrt{\frac{(f_{c_2} - f_{c_1})^2}{4(1-m^2)} + f_{c_1} f_{c_2}} - \frac{f_{c_2} - f_{c_1}}{2\sqrt{1-m^2}} \qquad \text{...(5)}$$

and

$$f_{\infty_2} = \sqrt{\frac{(f_{c_2} - f_{c_1})^2}{4(1-m^2)} + f_{c_1} f_{c_2}} + \frac{f_{c_2} - f_{c_1}}{2\sqrt{1-m^2}} \qquad \text{...(6)}$$

Subtracting equation (5) from equation (6), we get

$$f_{\infty_2} - f_{\infty_1} = \frac{f_{c_2} - f_{c_1}}{\sqrt{1-m^2}}$$

Solving for $m$, then we get

$$\boxed{m = \sqrt{1 - \left(\frac{f_{c_2} - f_{c_1}}{f_{\infty_2} - f_{\infty_1}}\right)^2}}$$

When multiplying equations (5) and (6), then we get

$$f_{\infty_1} \cdot f_{\infty_2} = \left[\frac{(f_{c_2} - f_{c_1})^2}{4(1-m^2)} + f_{c_1} f_{c_2}\right] - \left[\frac{(f_{c_2} - f_{c_1})^2}{4(1-m^2)}\right]$$

$$= f_{c_1} f_{c_2}$$

$$= f_0^2$$

or
$$\boxed{f_o = \sqrt{f_{\infty_1} \cdot f_{\infty_2}}}$$

The $m$-derived $T$-network band pass filter, arrange as $\pi$-network may be split into two half networks and used as terminating half networks.

**Note: If $m$ = 0.6, then satisfactory impedance matching conditions are maintained over the pass band.**

## 9.11. COMPOSITE FILTERS

We know that m-derived filters are designed in order to overcome the drawbacks of a constant-$K$ type filters. But the major drawback of m-derived filter is that, the attenuation in stop band drastically reduces after $f_\infty$ in low pass filter and before $f_\infty$ in high pass filter. In order to obtain best results, a composite filter is used. Such a filter consists constant-$K$, $m$-derived sections and terminating half $L$-networks as shown in Fig. 9.34. But, while connecting the different sections, there should be proper impedance matching between sections. For obtaining it, the different sections are terminated with two terminating half-section with m = 0.6.

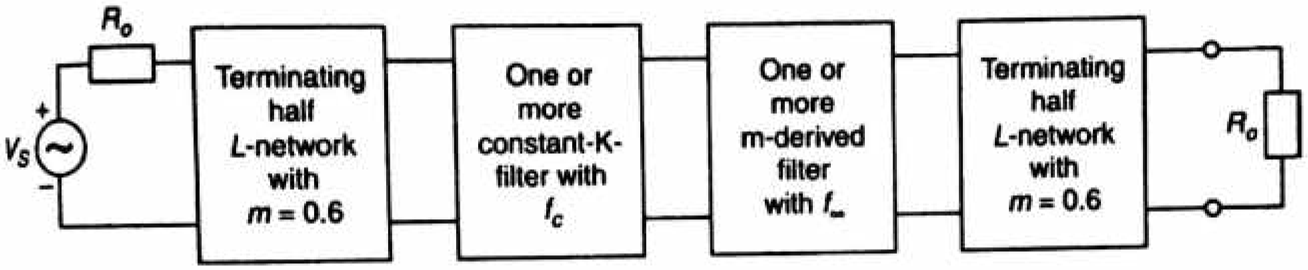



**Fig. 9.34.** Block diagram representation of a composite filter.

**In Composite Filter:**

(a) **Terminating half sections :** Provides proper impedance matching.

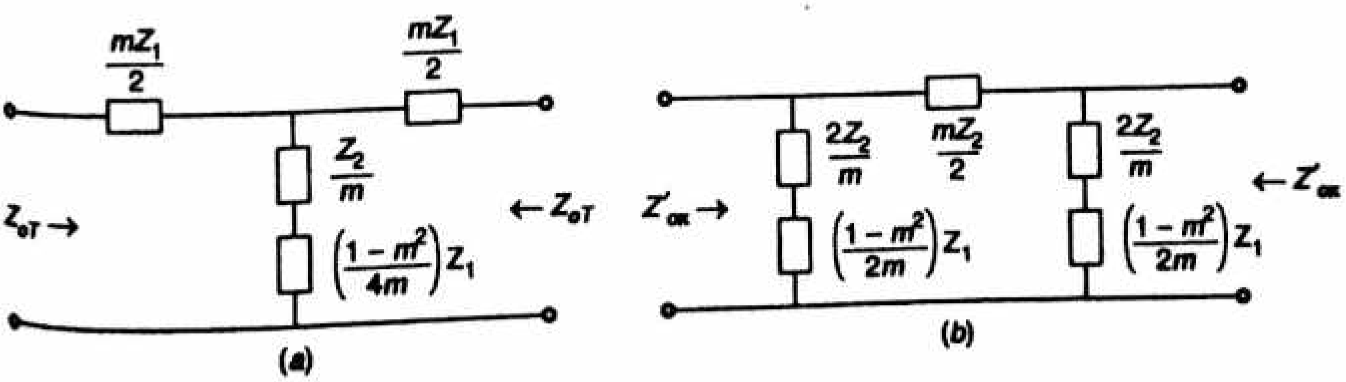
(b) **Constant-K-networks :** To give desired cut-off frequency $f_c$. Usually only one section is sufficient. But the number of sections depend on the attenuation required.

(c) **m-derived networks :** To give infinite attenuation at frequency $f_\infty$ close to $f_c$. The number of infinite attenuation frequencies decide the required number of $m$-derived sections. One section is required for $m$ = 0.3 to 0.35. $m$-derived sections are preferred, when a rapid rise in the attenuation is required.

The proper impedance matching in a pass band is obtained by using m-derived half sections. $m$-derived half sections are obtained by splitting $m$-derived $T$ or $\pi$ section centrally.

$m$-derived $T$-network filter, corresponding $\pi$-network filter and corresponding its half L-sections are shown in Fig. 9.35.

The characteristic impedances of the $m$-derived $T$-network filter at each end is $Z_{oT}$, for $\pi$-network is $Z'_{o\pi}$. The image impedances of these half networks seen from the two ends are $Z_{oT}$ and $Z'_{o\pi}$.



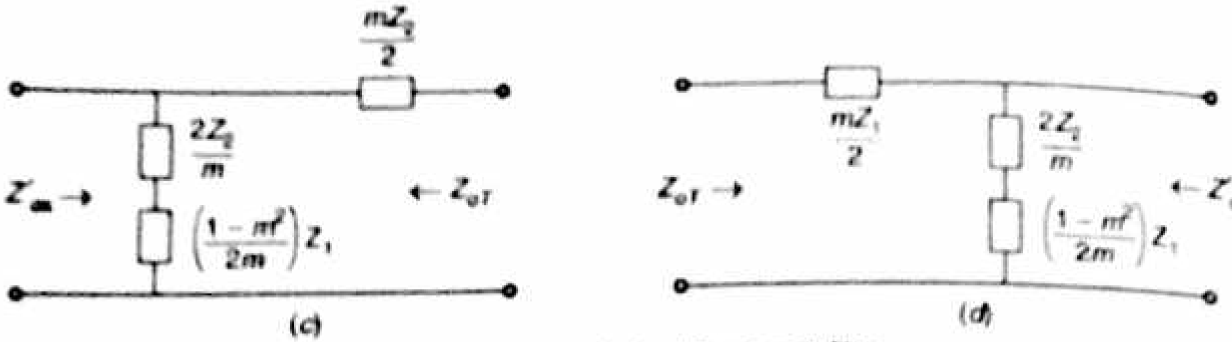



**Fig. 9.35.** (*a*) m-derived T-network filter
(*b*) corresponding π-network filter
(*c*) and (*d*) corresponding half *L*-sections.

Now, we derive an expression for the characteristic impedance of the m-derived T-network filter as shown in Fig. 9.35 (*a*).

Let $Z_1'$ and $Z_2'$ represent total series and shunt impedance respectively of the network of fig. 9.35 (*b*), then

$$Z_1' = mZ_1 \text{ and } Z_2' = \frac{Z_2}{m} + \left(\frac{1-m^2}{4m}\right) Z_1$$

Since, $$Z_{oT}\, Z'_{o\pi} = R_o^2 = Z_1' \; Z_2'$$

and $$Z_{oT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

then $$Z'_{o\pi} = \frac{Z'_1 Z'_2}{Z_{oT}} = \frac{(mZ_1)\left[\frac{Z_2}{m} + \left(\frac{1-m^2}{4m}\right) Z_1\right]}{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}}$$

$$= \frac{Z_1 Z_2 \left[1 + (1-m^2)\frac{Z_1}{4Z_2}\right]}{\sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}}$$

or $$Z'_{o\pi} = \frac{R_o\left[1 + (1-m^2)\frac{Z_1}{4Z_2}\right]}{\sqrt{\left(1 + \frac{Z_1}{4Z_2}\right)}} \qquad (\text{Since, } Z_1 Z_2 = R_o^2) \qquad ...(1)$$

Now, $$Z_{oT}\, Z_{o\pi} = R_o^2$$

$\therefore$ $$Z_{o\pi} = \frac{R_o^2}{Z_{oT}} = \frac{R_o^2}{\sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}} \qquad (\text{Since, } Z_1 Z_2 = R_o^2)$$

or $$Z_{o\pi} = \frac{R_o}{\sqrt{1 + \frac{Z_1}{4Z_2}}} \qquad ...(2)$$

From equations (1) and (2), we get

$$\boxed{Z'_{o\pi} = Z_{o\pi}\left[1+(1-m^2)\frac{Z_1}{4Z_2}\right]} \quad ...(3)$$

Equation (3), represents the required expression for the characteristics impedance of the half L-network $Z'_{o\pi}$ in terms of the characteristic impedance of the $\pi$-network filter $Z_{o\pi}$.

**(a) For Low-pass Filter**

We have, $\qquad Z_1 = j\omega L \quad$ and $\quad Z_2 = \dfrac{1}{j\omega C}$

Putting the values of $Z_1$ and $Z_2$ in equation (1), we get

$$Z'_{o\pi} = \frac{R_o\left[1-(1-m^2)\dfrac{\omega^2 LC}{4}\right]}{\sqrt{1-\dfrac{\omega^2 LC}{4}}} \qquad (\because j^2 = -1)$$

Since, $\qquad f_c = \dfrac{1}{\pi\sqrt{LC}} \quad$ and $\quad \omega = 2\pi f$

Therefore,
$$Z'_{o\pi} = \frac{R_o\left[1-(1-m^2)\left(\dfrac{f}{f_c}\right)^2\right]}{\sqrt{1-\left(\dfrac{f}{f_c}\right)^2}}$$

**(b) For High-Pass Filter**

We have, $\qquad Z_1 = \dfrac{1}{j\omega C}$ and $Z_2 = j\omega L$

Putting the values of $Z_1$ and $Z_2$ in equation (1), we get

$$Z'_{o\pi} = \frac{R_o\left[1-(1-m^2)\dfrac{1}{\omega^2 LC}\right]}{\sqrt{1-\dfrac{1}{\omega^2 LC}}}$$

or
$$Z'_{o\pi} = \frac{R_o\left[1-(1-m^2)\left(\dfrac{f_c}{f}\right)^2\right]}{\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}} \qquad \left(\text{Since, } f_c = \frac{1}{\pi\sqrt{LC}} \text{ and } \omega = 2\pi f\right)$$

From equation of $Z'_{o\pi}$ it is clear that $Z'_{o\pi}$ remains practically constant at the value of $R_o$ for $m = 0.6$ upto $f = 0.85\, f_c$ for low pass filter and upto $f = 1.18\, f_c$ for high pass filter. Beyond this frequency, $Z'_{o\pi}$ rapidly approaches infinity. Thus, if $m = 0.6$ is selected then the filter is terminated accurately into pure resistance. That means there is no mismatching and a filter is properly terminated.

The representation of the $m$-derived $T$-network terminated both sides by half $L$-networks is shown as Fig. 9.36.

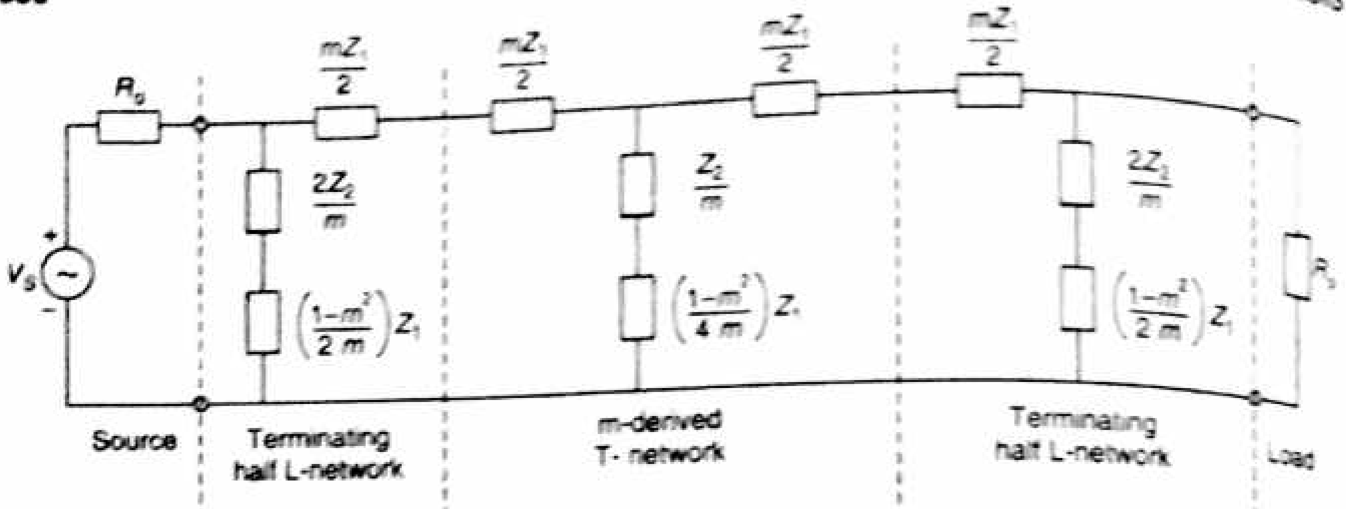



Fig. 9.36. Representation of the m-derived T-network terminated both sides by half L-networks.

### 9.11.1. Design Procedure of Composite Filters

The design procedure of a composite filters are as follows:

**Step-I:** First, design a constant-*K* type *T*-network filter for required cut-off frequency $f_c$ and nominal design impedance $R_o$.

**Step-II:** Then *m*-derived network is designed on the basis of T-network. The value of *m* is selected for the desired value of frequencies of $f_c$ and $f_\infty$.

**Step-III:** The m-derived *T*-network is connected in tandem with constant-*K* type *T*-network and terminated at each end by half *L*-network with $m = 0.6$.

Then we find a network having a constant design impedance $R_o$, cut-off frequency $f_c$ and infinite attenuation at $f_\infty$.

**Example 9.13.** *Design a composite low pass filter using T-network which is to be terminated in 400 Ω resistance. It must have a cut-off frequency of 800 Hz with very high attenuation at 865 Hz, 1000 Hz and at ∞ Hz.*

**Solution.**

**Step-I: To design the constant-*K* low pass filter.**

Given, $R_o = 400\ \Omega, f_c = 800$ Hz

For constant-*K* low pass filter, we have

$$f_c = \frac{1}{\pi\sqrt{LC}} \quad \text{and} \quad R_o = \sqrt{\frac{L}{C}}$$

From these equations, we get

$$L = \frac{R_o}{\pi f_c} \quad \text{and} \quad C = \frac{1}{\pi R_o f_c}$$

$$\therefore \quad L = \frac{400}{3.14 \times 800} = 159 \text{ mH}, \quad \text{or} \quad \frac{L}{2} = \frac{159 \times 10^{-3}}{2} = 79.5 \text{ mH}$$

and

$$C = \frac{1}{3.14 \times 400 \times 800} = 0.994\ \mu\text{F}$$

This constant-*K* low pass filter meets the specification for high attenuation at infinity ($f_\infty = \infty$ Hz).

**Step-II: To design m-derived *T*-network low pass filter for $f_\infty = 865$ Hz and $f_c = 800$ Hz**

We know that,

$$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}$$

$$\therefore \qquad m = \sqrt{1-\left(\frac{800}{865}\right)^2} = 0.38$$

Hence the component values of this filter are

$$\frac{mL}{2} = \frac{0.38 \times 159 \times 10^{-3}}{2} = 30.21 \text{ mH}$$

$$\left(\frac{1-m^2}{4m}\right)L = \frac{[1-(0.38)^2] \times 159 \times 10^{-3}}{4 \times 0.38} = 89.5 \text{ mH}$$

$$mC = 0.38 \times 0.994 \times 10^{-6} = 0.378\ \mu\text{F}$$

**Step-III: To design *m*-derived terminating half *L*-networks for $f_\infty = 1000$ Hz and $f_c = 800$ Hz.**

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} = \sqrt{1-\left(\frac{800}{1000}\right)^2} = 0.6$$

This value of *m* will be appropriate for use in the terminating half L-networks. The values of component of these sections are:

$$\frac{mL}{2} = \frac{0.6 \times 159 \times 10^{-3}}{2} = 47.4 \text{ mH}$$

$$\left(\frac{1-m^2}{2m}\right)L = \frac{\left[1-(0.6)^2\right] \times 159 \times 10^{-3}}{2 \times 0.6} = 86 \text{ mH}$$

$$\frac{mC}{2} = \frac{0.6 \times 0.994 \times 10^{-6}}{2} = 0.298\ \mu\text{F}.$$

**Step-IV: To obtain complete design of the composite filter. The composite filter is shown in Fig. E 9.13 (*a*).**

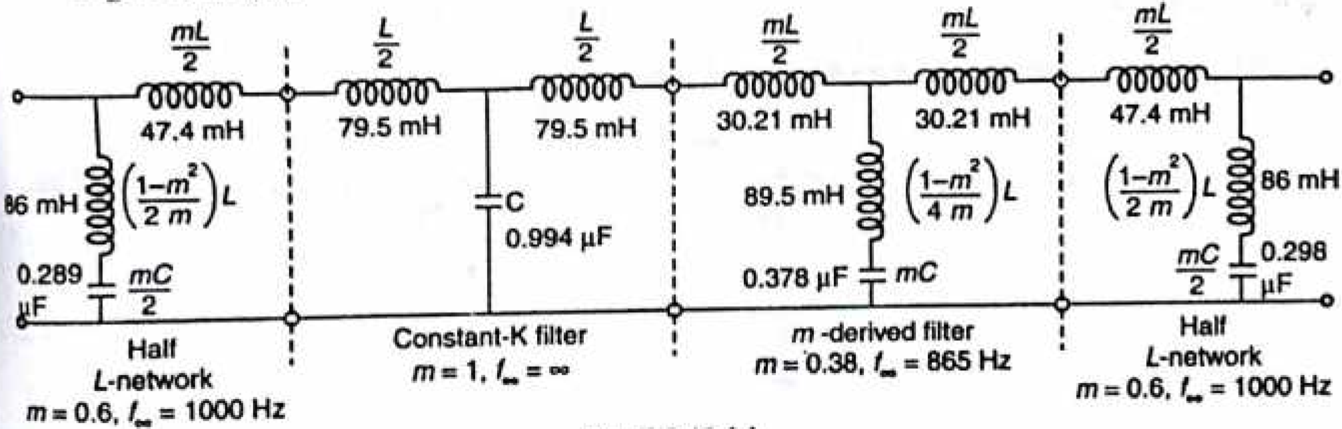



Fig. E 9.13 (*a*)

The simplified version of composite filter is shown in Fig. E 9.13 (*b*)

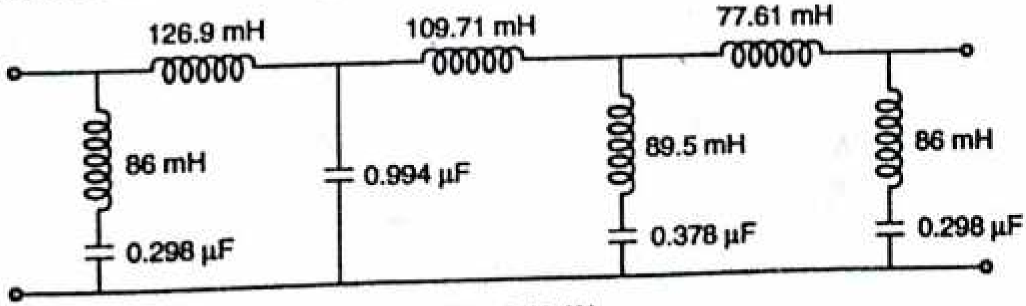



Fig. 9.13 (*b*)

## 9.12. LIMITATIONS OR DRAWBACKS OF PASSIVE FILTERS

The passive filters have many drawbacks some of these are as follow

(*i*) They require an external amplifier to provide suitable gain.

(*ii*) For making a composite filter, several stages are cascaded together. During cascading in order to prevent loading, isolation amplifiers are needed.

(*iii*) At low frequencies the use of inductor is undesirable, as inductors are bulky and costly.

(*iv*) High range of quality factor (or $Q$-factor) is not possible.

## 9.13. ACTIVE FILTERS

Active filters are a class of frequency selective circuits in which resistances, capacitances and active elements (*i.e.*, transistors, op-amps) are used. The design of active filters is based on the filter transfer function which satisfies certain specifications.

**The advantages of active filters are as follow:**

(*i*) They provide good gain and excellent isolation properties, *i.e.*, high input and low output impedances.

(*ii*) They eliminate the need for inductors which are large, bulky (heavy), costly, non-linear (some-times) generate stray magnetic fields and may dissipate considerable power.

(*iii*) High range of quality factor is possible.

(*iv*) They are compatible with integrated circuits, due to the absence of inductors.

(*v*) Reduction in power consumption.

(*vi*) They can be designed and tuned independently with minimum interaction, due to its excellent isolation property.

### 9.13.1. First Order Active Low Pass Filter

The circuit of a first order active low pass filter is shown in Fig. 9.37. As we know that the input impedance of an operation amplifier is very high (*i.e.*, infinite for ideal op-amps), therefore the input current is almost zero and hence the current through $R_1$ and $R_2$ are nearly equal.

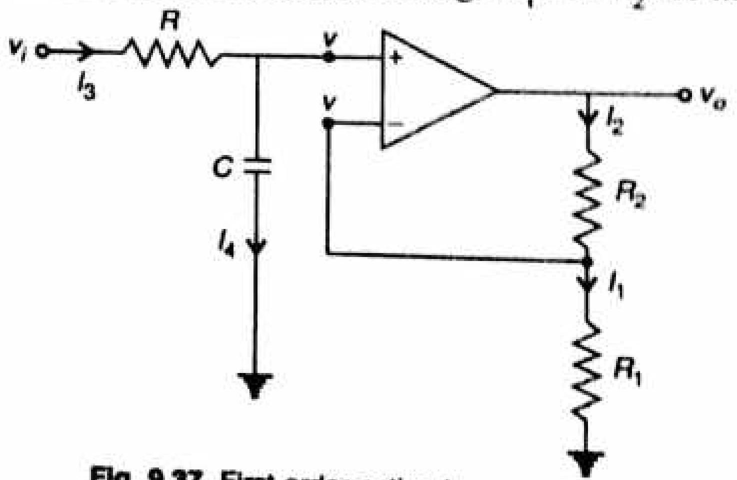



**Fig. 9.37.** First order active low pass filter.

Since,
$$I_1 \simeq I_2$$

$\therefore$
$$\frac{v-0}{R_1} = \frac{v_o - v}{R_2}$$

$$\frac{R_2}{R_1} v + v = v_o$$

or
$$v = \frac{v_o}{1+\dfrac{R_2}{R_1}} \quad ...(1)$$

From figure, we have

$$I_3 = I_4$$

or
$$\frac{v_i - v}{R} = \frac{v - 0}{\left(\frac{1}{sC}\right)} \qquad \left(\because X_c = \frac{1}{sC}\right)$$

$$v_i\left(\frac{R}{sC}\right) = v + \frac{v}{RCs}$$

or
$$v = \frac{v_i}{1 + RCs} \qquad ...(2)$$

From equations (1) and (2), we get

$$\frac{v_o}{1 + \frac{R_2}{R_1}} = \frac{v_i}{1 + RCs}$$

or
$$\frac{v_o}{v_i} = \frac{1 + \frac{R_2}{R_1}}{1 + RCs} \qquad ...(3)$$

From equation (1), the gain of the op-amps is given by

$$\boxed{A_o = \frac{v_o}{v} = 1 + \frac{R_2}{R_1}} \qquad ...(4)$$

From equations (3) and (4), the transfer functions of first order low pass filter is given by

$$\boxed{H(s) = \frac{v_o}{v_i} = \frac{A_o}{1 + RCs} = \frac{A_o}{1 + \left(\frac{s}{\omega_o}\right)}}$$

where $\omega_o = \frac{1}{RC}$, the cut-off frequency.

Therefore, for given cut-off frequency $\omega_o$ and assumed value of $C$, the value of $R$ can be evaluated. Similarly, for arbitrary gain $A_o$ and some assumed value of $R_1$, the value of $R_2$ can be evaluated and hence first order active low-pass filter can be designed.

### 9.13.2. First Order Active High Pass Filter

The circuit of a first order active high pass filter is shown in Fig. 9.38.

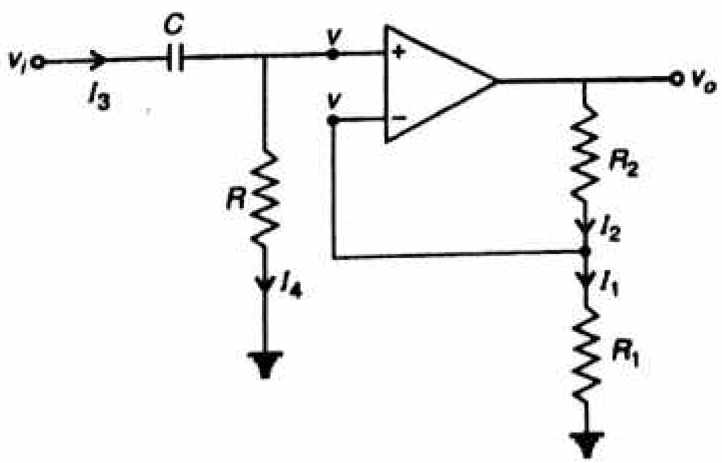



Fig. 9.38.

Since, $$I_1 = I_2$$

$\therefore$ $$\frac{v-0}{R_1} = \frac{v_0 - v}{R_2}$$

$$\frac{R_2}{R_1}v + v = v_0$$

or $$v = \frac{v_0}{1+\frac{R_2}{R_1}} \quad ...(1)$$

From figure, we have

$$I_3 = I_4$$

or $$\frac{v_i - v}{\left(\frac{1}{sC}\right)} = \frac{v-0}{R}$$

$$v_i = \frac{v}{RCs} + v$$

or $$v = \frac{v_i}{1+\frac{1}{RCs}} \quad ...(2)$$

From equations (1) and (2), we get

$$\frac{v_o}{1+\frac{R_2}{R_1}} = \frac{v_i}{1+\frac{1}{RCs}}$$

or $$\frac{v_o}{v_i} = \frac{1+\frac{R_2}{R_1}}{1+\frac{2}{RCs}} \quad ...(3)$$

From equation (1), the gain of the op-amp is given by

$$A_0 = \frac{v_0}{v} = 1 + \frac{R_2}{R_1} \quad ...(4)$$

From equations (3) and (4), the transfer function of first order active high pass filter is given by

$$\boxed{H(s) = \frac{v_0}{v_i} = \frac{A_0}{1+\frac{1}{RCs}} = \frac{A_0}{1+\left(\frac{\omega_0}{s}\right)}}$$

where $\omega_o = \frac{1}{RC}$, the cut-off frequency

**Note:** The first order active high pass filter is obtained by inter changing $R$ and $C$ of first order active low pass filter. The transfer function is given by mathematically applying the transformation

$$\left.\frac{s}{\omega_o}\right|_{\text{Low pass filter}} \longrightarrow \left.\frac{1}{\left(\frac{s}{\omega_o}\right)}\right|_{\text{High pass filter}}$$

### 9.13.3. Active Band Pass Filter

The active band-pass filter can be obtained by cascading active low-pass filter and active high-pass filter as shown in Fig. 9.39.

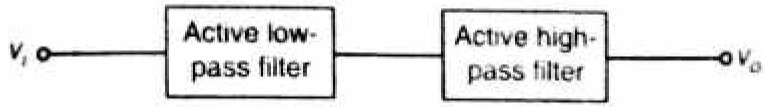
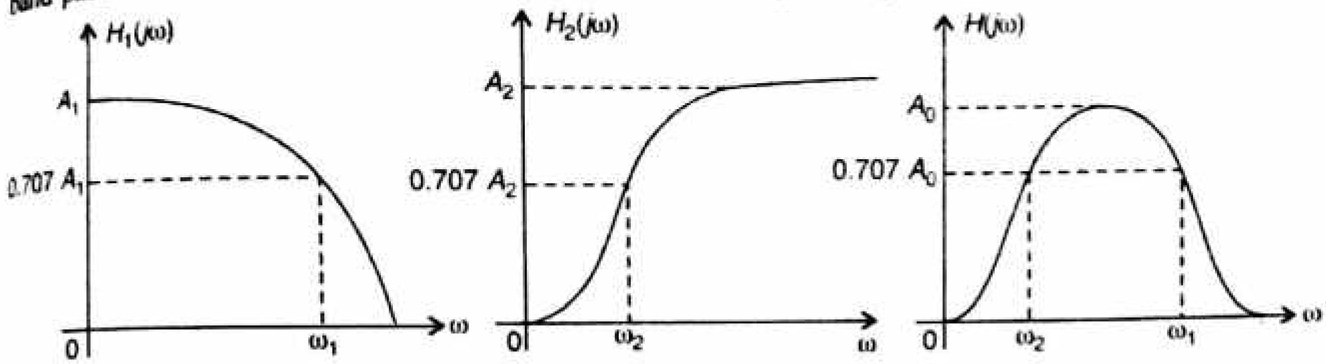



**Fig. 9.39.** Block diagram of an active band-pass filter.

The practical frequency responses of the active low-pass filter, active high-pass filter and active band-pass filter are shown in Fig. 9.40 (*a*), (*b*) and (*c*) respectively.




(*a*) Frequency response of active low-pass filter

(*b*) Frequency response of active high-pass filter

(*c*) Frequency response of active band-pass filter

**Fig. 9.40.** Frequency responses of filters

### 9.13.4. Active Band-Stop Filter

The active band-stop filter can be obtained by parallel combination of active low-pass and active high-pass filters as shown in Fig. 9.41.

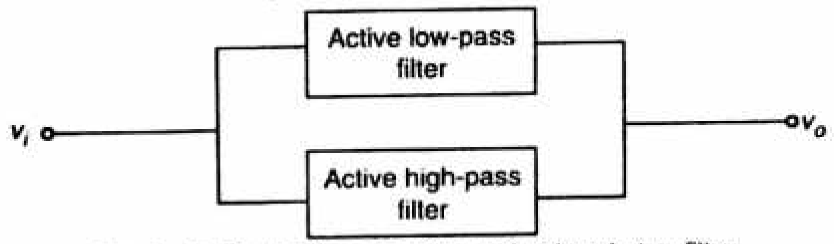
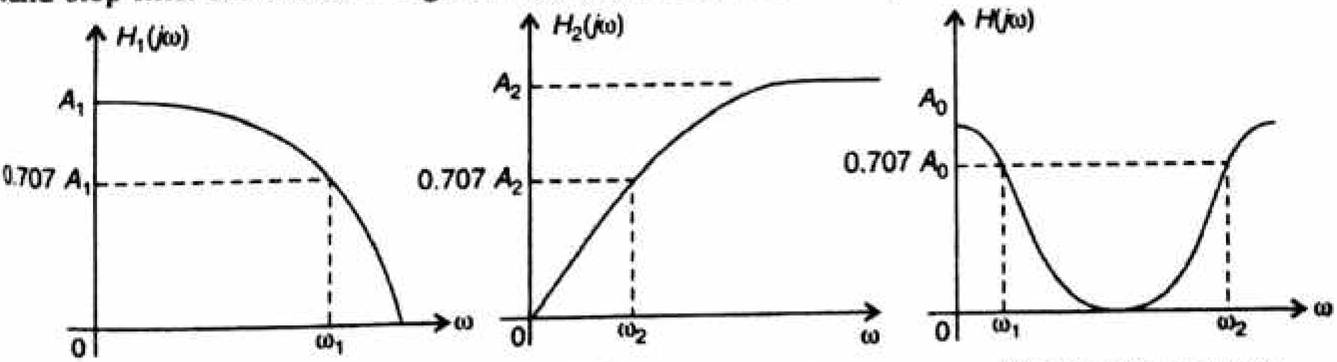



**Fig. 9.41.** Block diagram of an active band stop filter.

The practical frequency responses of the active low-pass filter, active high-pass filter and active band-stop filter are shown in Fig. 9.42 (*a*), (*b*) and (*c*) respectively.




(*a*) Frequency response of active low-pass filter

(*b*) Frequency response of active high-pass filter

(*c*) Frequency response of active band-stop filter

**Fig. 9.42.** Frequency responses of filters

## 9.14 SOLVED EXAMPLES ON FILTERS

**Example 9.14.** *A T-section low pass filter has series inductance 80 mH and shunt capacitance 0.022 μF. Determine the cut-off frequency and nominal design impedance. Also design an equivalent π-section.* **(UPTU-2006)**

**Solution.**

**Step-I: To determine the cut-off frequency.**

The $T$-section low pass filter is shown as shown in Fig. E 9.14 ($a$).

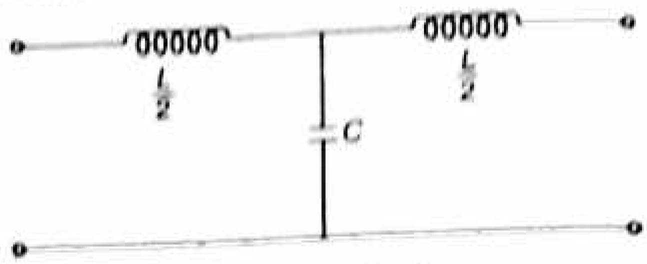



**Fig. E 9.14 ($a$)**

Given, series inductance, $\frac{L}{2} = 80$ mH

or $L = 160$ mH

Shunt capacitance, $C = 0.022\ \mu$F.

We know that cut-off frequency;

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

$$\therefore \quad f_c = \frac{1}{3.14\sqrt{(160\times10^{-3})\times(0.022\times10^{-6})}}$$

or $f_c =$ **5.37 kHz**

**Step-II: To determine the nominal design impedance.** The design (or characteristic) impedance is given by

$$R_o = \sqrt{\frac{L}{C}}$$

$$\therefore \quad R_o = \sqrt{\frac{160\times10^{-3}}{0.022\times10^{-6}}} = \mathbf{2.697\ k\Omega}$$

**Step-III: To design an equivalent-π section.**

The equivalent-π section of a $T$-section low pass filter is shown in Fig. E 9.14 ($b$)

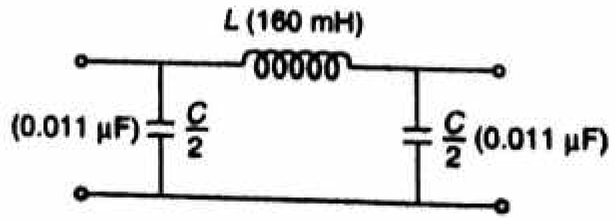



**Fig. E 9.14 (b)**

**Example 9.15.** *A series LCR type band stop filter has R = 1.5 kΩ, L = 140 mH and C = 300 pf. Find the (i) Resonant frequency and (ii) Bandwidth. Also find the cut-off frequencies.* **(UPTU-2006)**

**Solution.**

**Step-I: To determine resonant frequency.**

Given, $R = 1.5$ kΩ, $L = 140$ mH, $C = 300$ pf.

The resonant frequency is given by

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \quad f_r = \frac{1}{2 \times 3.14\sqrt{(140 \times 10^{-3}) \times (300 \times 10^{-12})}} = \mathbf{24.57\ kHz.}$$

**Step-II: To determine bandwidth.**

$$\text{Bandwidth} = \frac{R}{2\pi L} = \frac{1.5 \times 10^3}{2 \times 3.14 \times 140 \times 10^{-3}} = \mathbf{1.706\ kHz.}$$

**Step-III: To determine cut-off frequencies.**

Cut-off frequencies, $$f_{c_1}, f_{c_2} = f_r \pm \frac{\text{Bandwidth}}{2}$$

$$\therefore \quad f_{c_1} = 24.57 - \frac{1.706}{2} = \mathbf{23.72\ kHz}$$

and $$f_{c_2} = 24.57 + \frac{1.706}{2} = \mathbf{25.42\ kHz}$$

**Example 9.16.** *Design a T-section constant K-high pass filter having cut-off frequency of 10 kHz and design impedance of 600 Ω. Find its characteristic impedance and phase constant at 25 kHz.* **(UTPU-2006)**

**Solution.**

**Step-I: To design (or obtain parameters) of a *T*-section constant *K*-high pass filter.**

Given, $f_c = 10$ kHz, $R_o = 600\ \Omega$

The *T*-section constant K-high pass filter is shown in Fig. E 9.16.

We know that

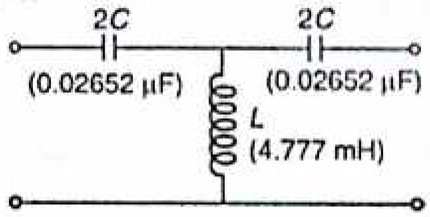



Fig. E9.16

$$R_0 = \sqrt{\frac{L}{C}} \quad \text{and} \quad f_c = \frac{1}{2\pi\sqrt{LC}}$$

From these equations, we get

$$L = \frac{R_o}{4\pi f_c} \quad \text{and} \quad C = \frac{1}{4\pi f_c R_o}$$

$$\therefore \quad L = \frac{600}{4 \times 3.14 \times 10 \times 10^3} = \mathbf{4.777\ mH}$$

and $$C = \frac{1}{4 \times 3.14 \times 10 \times 10^3 \times 600} = \mathbf{0.01326\ \mu F}$$

**Step-II: To determine the characteristic impedance and phase constant at 25 kHz.**

We know that at any frequency *f*, the characteristic impedance for T-section high pass filter is given by

$$Z_{oT} = R_o\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\therefore \quad Z_{oT} = 600\sqrt{1 - \left(\frac{10 \times 10^3}{25 \times 10^3}\right)^2} = \mathbf{545\ \Omega}$$

The phase constant is given by

$$\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right)$$

$$\therefore \quad \beta = 2\sin^{-1}\left(\frac{10\times10^3}{25\times10^3}\right) = \mathbf{47.2° = 0.82\ radian}$$

## THEORETICAL QUESTIONS

**9.1** Discuss the disadvantages of *k*-byte filters. How can these be overcome using m-derived filters? **(UPTU-2005-2006)**

**9.2** What is filter? Define pass band, stop band and cut-off frequency.

**9.3** What is constant K-filter? Why it is called proto type filter section?

**9.4** Derive the expression for cut-off frequency $f_c$ of proto type low pass filter.

**9.5** Derive the expression for cut-off frequency $f_c$ of proto type high pass filter.

**9.6** For band pass filter show that series and shunt arm resonant frequency is the geometric mean of two cut-off frequencies $f_{c_1}$ and $f_{c_2}$ deciding pass band.

**9.7** Define all the characteristics of filter networks.

**9.8** Define *m*-derived filters. Derive the expression of m for m-derived (*a*) low-pass, (*b*) high-pass (c) band-pass filters.

**9.9** Derive all the characteristics of constant-*K* (*a*) low-pass, (*b*) high-pass, (*c*) band-pass (*d*) band-stop filters.

**9.10** Define composite filters and describe the design procedure for the composite filters.

**9.11** What are limitations of passive filters?

**9.12** Define active filters. List advantages of active filters.

## NUMERICAL PROBLEMS

**9.1** Design constant-K type low pass filter as *T* and π-networks for characteristic impedance of 600 Ω and cut-off frequency of 1200 Hz. **[Ans.** $L$ = 159.15 mH, $C$ = 0.442 μF**]**

**9.2** Design a low pass filter as *T* and π-networks with $f_c$ = 2000 Hz to operate a terminated load resistance of 500 Ω. **[Ans.** $L$ = 79.6 mH, $C$ = 0.3189 μF**]**

**9.3** Design a high pass filter with cut-off frequency of 1000 Hz and load impedance of 600 Ω. At what frequency α will be 10 dB? **[ Ans:** $L$ = 47.74 mH, $C$ = 0.133 μF**]**

α = 1.15 Nepers.

**9.4** A π-section filter network consists of a series arm inductance 20 mH and two shunt arm capacitors of 0.16 μF each. Determine $f_c$ and attenuation and phase shift at 1.5 kHz. What is the value of nominal impedance in the pass band?

**[Ans.** $f_c$ = 3.98 kHz, $R_o$ = 250 Ω, α = 0, β = 0.773 rad**]**

**9.5** Design a proto type *HPF* having $R_o$ = 600 Ω and $f_c$ = 2 kHz.

**[Ans.** $L$ = 23.87 mH, $C$ = 0.0663 μF**]**

**9.6** Design a constant-*K* high pass π-section filter to work into $R_o$ = 500 Ω and $f_c$ = 1000 Hz. Determine (*a*) phase angle (β) at a $f$ = 1500 Hz (*b*) attenuation in Nepers at a $f$ = 0.9 kHz.

**[Ans.** $L$ = 39.8 mH, $C$ = 0.16 μF, β = – 83.6°, α = 0.9343 Nepers**]**