Bessel Filter Constants

1st Order

The minimum phase transfer function is

$$H(s) = \frac{1}{s+1}. \tag{1}$$

The corresponding amplitude response function in $j\omega$ is

$$H(j\omega) = \frac{1}{1+j\omega}. \tag{2}$$

The magnitude response is

$$|H(jw)| = \frac{1}{\sqrt{\omega^2+1}}. \tag{3}$$

The phase response is

$$\phi(\omega) = -\tan^{-1}(\omega). \tag{4}$$

The group delay is

$$\tau(\omega) = \frac{1}{1+\omega^2}. \tag{5}$$

2nd Order

The minimum phase transfer function is

$$H(s) = \frac{3}{s^2+3s+3}. \tag{6}$$

The corresponding amplitude response function in $j\omega$ is

$$H(j\omega) = \frac{3}{(3-\omega^2)+j3\omega}. \tag{7}$$

The magnitude response is

$$|H(jw)| = \frac{3}{\sqrt{\omega^4+3\omega^2+9}}. \tag{8}$$

The phase response is

$$\phi(\omega) = -\tan^{-1}\left(\frac{3\omega}{3-\omega^2}\right). \tag{9}$$

The group delay is

$$\tau(\omega) = \frac{9+3\omega^2}{9+3\omega^2+\omega^4}. \tag{10}$$

The 3dB frequency normalization constant is: 1.361654129

3rd Order

The minimum phase transfer function is

$$H(s) = \frac{15}{s^3 + 6s^2 + 15s + 15}. \tag{11}$$

The corresponding amplitude response function in $j\omega$ is

$$H(j\omega) = \frac{15}{(15 - 6\omega^2) + j(15\omega - \omega^3)}. \tag{12}$$

The magnitude response is

$$|H(jw)| = \frac{15}{\sqrt{\omega^6 + 6\omega^4 + 45\omega^2 + 225}}. \tag{13}$$

The phase response is

$$\phi(\omega) = -\tan^{-1}\left(\frac{15\omega - \omega^3}{15 - 6\omega^2}\right). \tag{14}$$

The group delay is

$$\tau(\omega) = \frac{225 + 45\omega^2 + 6\omega^4}{225 + 45\omega^2 + 6\omega^4 + \omega^6}. \tag{15}$$

The 3dB frequency normalization constant is: 1.755672389

4th Order

The minimum phase transfer function is

$$H(s) = \frac{105}{s^4 + 10s^3 + 45s^2 + 105s + 105}. \tag{16}$$

The corresponding amplitude response function in $j\omega$ is

$$H(j\omega) = \frac{105}{(\omega^4 - 45\omega^2 + 105) + j(105\omega - 10\omega^3)}. \tag{17}$$

The magnitude response is

$$|H(jw)| = \frac{105}{\sqrt{\omega^8 + 10\omega^6 + 135\omega^4 + 1575\omega^2 + 11025}}. \tag{18}$$

The phase response is

$$\phi(\omega) = -\tan^{-1}\left(\frac{105\omega - 10\omega^3}{105 - 45\omega^2 + \omega^4}\right). \tag{19}$$

The group delay is

$$\tau(\omega) = \frac{11025 + 1575\omega^2 + 135\omega^4 + 10\omega^6}{11025 + 1575\omega^2 + 135\omega^4 + 10\omega^6 + \omega^8}. \tag{20}$$

The 3dB frequency normalization constant is: 2.113917675

5th Order

The minimum phase transfer function is

$$H(s) = \frac{945}{s^5 + 15s^4 + 105s^3 + 420s^2 + 945s + 945}. \tag{21}$$

The corresponding amplitude response function in $j\omega$ is

$$H(j\omega) = \frac{945}{(945 - 420\omega^2 + 15\omega^4) + j(945\omega - 105\omega^3 + \omega^5)}. \tag{22}$$

The magnitude response is

$$|H(jw)| = \frac{945}{\sqrt{\omega^{10} + 15\omega^8 + 315\omega^6 + 6300\omega^4 + 99225\omega^2 + 893025}}. \tag{23}$$

The phase response is

$$\phi(\omega) = -\tan^{-1}\left(\frac{945\omega - 105\omega^3 + \omega^5}{945 - 420\omega^2 + 15\omega^4}\right) \tag{24}$$

The group delay is

$$\tau(\omega) = \frac{893025 + 99225\omega^2 + 6300\omega^4 + 315\omega^6 + 15\omega^8}{893025 + 99225\omega^2 + 6300\omega^4 + 315\omega^6 + 15\omega^8 + \omega^{10}}. \tag{25}$$

The 3dB frequency normalization constant is: 2.427410702

6th Order

The minimum phase transfer function is

$$H(s) = \frac{10395}{s^6 + 21s^5 + 210s^4 + 1260s^3 + 4725s^2 + 10395s + 10395}. \tag{26}$$

The corresponding amplitude response function is

$$H(j\omega) = \frac{10395}{(10395 - 4725\omega^2 + 210\omega^4 - \omega^6) + j\omega(10395 - 1260\omega^2 + 21\omega^4)} \quad (27)$$

The magnitude response is

$$|H(j\omega)| = \frac{10395}{\sqrt{\omega^{12} + 21\omega^{10} + 630\omega^8 + 18900\omega^6 + 496125\omega^4 + 9823275\omega^2 + 108056025}} \quad (28)$$

The phase response is

$$\phi(\omega) = -\tan^{-1}\left(\frac{10395\omega - 1260\omega^3 + 21\omega^5}{10395 - 4725\omega^2 + 210\omega^4 - \omega^6}\right) \quad (29)$$

The group delay is

$$\tau(\omega) = \frac{108056025 + 9823275\omega^2 + 496125\omega^4 + 18900\omega^6 + 630\omega^8 + 21\omega^{10}}{108056025 + 9823275\omega^2 + 496125\omega^4 + 18900\omega^6 + 630\omega^8 + 21\omega^{10} + \omega^{12}} \quad (30)$$

The 3dB frequency normalization constant is: 2.703395061